

**A**NALYTICAL MECHANICS provides a detailed introduction to the key analytical techniques of classical mechanics, one of the cornerstones of physics. It deals with all the important subjects encountered in an undergraduate course and prepares the reader thoroughly for further study at the graduate level.

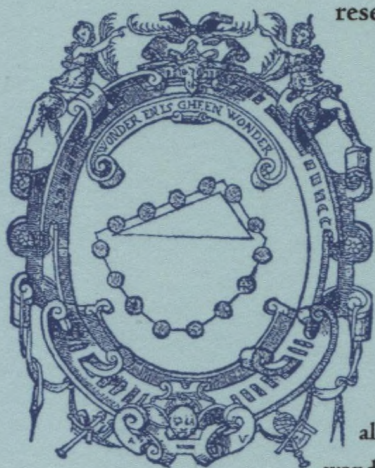
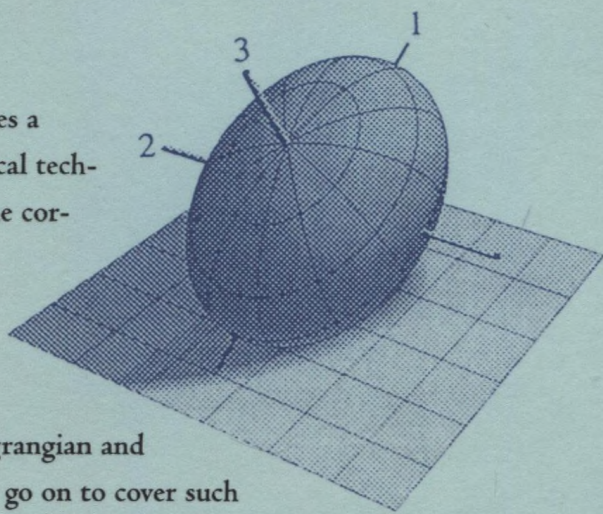
The authors set out the fundamentals of Lagrangian and Hamiltonian mechanics early on in the book and go on to cover such topics as linear oscillators, planetary orbits, rigid-body motion, small vibrations, non-linear dynamics, chaos, and special relativity. A special feature is the inclusion of many “e-mail questions,” which are intended to facilitate dialogue between the student and instructor.

Many worked examples are given, and there are 250 homework exercises to help students gain confidence and proficiency in problem solving. It is an ideal textbook for undergraduate courses in classical mechanics and provides a sound foundation for graduate study.

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The figure at left is from the title page of the 1605 book by Stevinus on mechanics, written long before Newton. The endless chain cannot exhibit perpetual motion. From this symmetry principle, Stevinus deduced the parallelogram of forces, a crucial discovery in mechanics. “Wonder en is gheen wonder” is 17th century Dutch and means “every enlightening progress made in science is accompanied with a certain feeling of disillusionment” (translation by Ernst Mach).





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