

Contents

Preface to the Second Edition	v
Preface to the First Edition	vii
Notation	xiii
Chapter 1	
Special Equations	1
§ 1. Differential Equations Invariant under Groups of Symmetries	1
§ 2. Resolution of Singularities of Differential Equations	9
§ 3. Implicit Equations	14
§ 4. Normal Form of an Implicit Differential Equation in the Neighborhood of a Regular Singular Point	25
§ 5. The Stationary Schrödinger Equation	31
§ 6. Geometry of a Second-Order Differential Equation and Geometry of a Pair of Direction Fields in Three-Dimensional Space	43
Chapter 2	
First-Order Partial Differential Equations	59
§ 7. Linear and Quasilinear First-Order Partial Differential Equations	59
§ 8. The Nonlinear First-Order Partial Differential Equation	68
§ 9. A Theorem of Frobenius	86
Chapter 3	
Structural Stability	89
§ 10. The Notion of Structural Stability	89
§ 11. Differential Equations on the Torus	97
§ 12. Analytic Reduction of Analytic Circle Diffeomorphisms to a Rotation	114
§ 13. Introduction to the Hyperbolic Theory	121
§ 14. Anosov Systems	128
§ 15. Structurally Stable Systems Are Not Everywhere Dense	141
Chapter 4	
Perturbation Theory	144
§ 16. The Averaging Method	144
§ 17. Averaging in Single-Frequency Systems	148
§ 18. Averaging in Systems with Several Frequencies	153

§ 19. Averaging in Hamiltonian Systems	164
§ 20. Adiabatic Invariants	168
§ 21. Averaging in Seifert's Foliation	173
 Chapter 5	
Normal Forms	180
§ 22. Formal Reduction to Linear Normal Forms	180
§ 23. The Case of Resonance	184
§ 24. Poincaré and Siegel Domains	187
§ 25. Normal Form of a Mapping in the Neighborhood of a Fixed Point	192
§ 26. Normal Form of an Equation with Periodic Coefficients	195
§ 27. Normal Form of the Neighborhood of an Elliptic Curve	202
§ 28. Proof of Siegel's Theorem	214
 Chapter 6	
Local Bifurcation Theory	222
§ 29. Families and Deformations	222
§ 30. Matrices Depending on Parameters and Singularities of the Decrement Diagram	238
§ 31. Bifurcations of Singular Points of a Vector Field	260
§ 32. Versal Deformations of Phase Portraits	265
§ 33. Loss of Stability of an Equilibrium Position	270
§ 34. Loss of Stability of Self-Sustained Oscillations	286
§ 35. Versal Deformations of Equivariant Vector Fields on the Plane	302
§ 36. Metamorphoses of the Topology at Resonances	323
§ 37. Classification of Singular Points	337
Samples of Examination Problems	342
Index	345