

This classroom-tested textbook is an introduction to probability theory with the right balance between mathematical precision, probabilistic intuition, and concrete applications. *Introduction to Probability* covers the material precisely, while avoiding excessive technical details. Intended for students with a calculus background, the text teaches not only the nuts and bolts of probability theory and how to solve specific problems, but also why the methods of solution work.

FEATURES

- Presented in full color and written in an accessible way, the text provides a comprehensive and well-balanced introduction to probability.
- Pedagogical features include numerous examples to illustrate concepts and theory, over 600 exercises of varying levels, and separate *Finer Points* sections for technical details.

"The authors have carefully chosen a set of core topics, resisting the temptation to overload the reader. They tie it all together with a coherent philosophy. Knowing the authors' work, I would expect nothing less. I predict that this text will become the standard for beginning probability courses."

– Carl Mueller, *University of Rochester*

"This is an excellent book written by three active researchers in probability that combines both solid mathematics and the distinctive style of thinking needed for modeling random systems. It also has a great collection of problems. I expect it to become a standard textbook for undergraduate probability courses at least in the US."

– Gregory F. Lawler, *University of Chicago*



Online Resources
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▼ Instructor's Manual with solutions to selected problems

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