Statistics

Emphasizing the use of WinBUGS and R to analyze real data, **Bayesian Ideas and Data Analysis: An Introduction for Scientists and Statisticians** presents statistical tools to address scientific questions. It highlights foundational issues in statistics, the importance of making accurate predictions, and the need for scientists and statisticians to collaborate in analyzing data.

The first five chapters of the book contain core material that spans basic Bayesian ideas, calculations, and inference, including modeling one and two sample data from traditional sampling models. The text then covers Monte Carlo methods, such as Markov chain Monte Carlo (MCMC) simulation. After discussing linear structures in regression, it presents binomial regression, normal regression, analysis of variance, and Poisson regression, before extending these methods to handle correlated data. The authors also examine survival analysis and binary diagnostic testing. The last chapter on nonparametric inference explores density estimation and flexible regression modeling of mean functions.

Features

- Covers a large number of statistical models
- Emphasizes elicitation of real prior information
- Explores numerical approximations via simulation
- Uses WinBUGS and R for computations
- Includes numerous exercises and real-world examples
- Provides data, programming code, and other materials on the book's website

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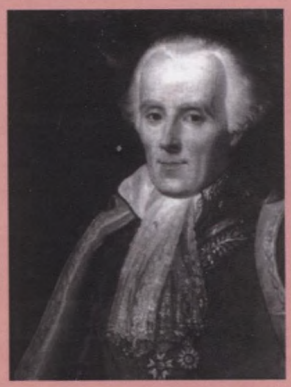

Pierre-Simon Laplace

CRC Press
Taylor & Francis Group
an **informa** business
www.crcpress.com

6000 Broken Sound Parkway, NW
Suite 300, Boca Raton, FL 33487
270 Madison Avenue
New York, NY 10016
2 Park Square, Milton Park
Abingdon, Oxon OX14 4RN, UK

K10199

ISBN: 978-1-4398-0354-7
90000
9 781439 803547

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