CONTENTS

Preface	ix
Prologue: Logical Roots of the Digital Age	1
1. An Ancient Tradition	5
1.1. Reduction to the Evident	5
1.2. Aristotle's Deductive Logic	7
1.3. Infinity and Incommensurability	16
1.4. Deductive and Marginal Notions of Truth	21
2. The Emergence of Foundational Study	29
2.1. In Search of the Roots of Formal Computation	31
2.2. Grassmann's Formalization of Calculation	40
2.3. Peano: The Logic of Grassmann's Formal Proofs	50
2.4. Axiomatic Geometry	57
2.5. Real Numbers	69
3. The Algebraic Tradition of Logic	81
3.1. Boole's Logical Algebra	81
3.2. Schröder's Algebraic Logic	83
3.3. Skolem's Combinatorics of Deduction	86
4. Frege's Discovery of Formal Reasoning	94
4.1. A Formula Language of Pure Thinking	94
4.2. Inference to Generality	110
4.3. Equality and Extensionality	112
4.4. Frege's Successes and Failures	117

viii • Contents

5. Russell: Adding Quantifiers to Peano's Logic	128
5.1. Axiomatic Logic	128
5.2. The Rediscovery of Frege's Generality	131
5.3. Russell's Failures	137
6. The Point of Constructivity	140
6.1. Skolem's Finitism	140
6.2. Stricter Than Skolem: Wittgenstein and His Students	151
6.3. The Point of Intuitionistic Geometry	167
6.4. Intuitionistic Logic in the 1920s	173
7. The Göttingers	185
7.1. Hilbert's Program and Its Programmers	185
7.2. Logic in Göttingen	191
7.3. The Situation in Foundational Research	
around 1930	210
8. Gödel's Theorem: An End and a Beginning	230
8.1. How Gödel Found His Theorem	230
8.2. Consequences of Gödel's Theorem	243
8.3. Two "Berliners"	248
9. The Perfection of Pure Logic	255
9.1. Natural Deduction	256
9.2. Sequent Calculus	286
9.3. Logical Calculi and Their Applications	303
10. The Problem of Consistency	318
10.1. What Does a Consistency Proof Prove?	319
10.2. Gentzen's Original Proof of Consistency	326
10.3. Bar Induction: A Hidden Element in the	
Consistency Proof	343
References	353
Index	373