

CONTENTS

PREFACE	ix	3. Definition of topos	84
PREFACE TO SECOND EDITION	xiv	4. First examples	85
PREFACE TO DOVER EDITION	xv	5. Bundles and sheaves	88
PROSPECTUS	1	6. Monoid actions	100
CHAPTER 1.		7. Power objects	103
MATHEMATICS = SET THEORY?	6	8. Ω and comprehension	107
1. Set theory	6	CHAPTER 5.	
2. Foundations of mathematics	13	TOPOS STRUCTURE: FIRST STEPS	109
3. Mathematics as set theory	14	1. Monics equalise	109
CHAPTER 2.		2. Images of arrows	110
WHAT CATEGORIES ARE	17	3. Fundamental facts	114
1. Functions are sets?	17	4. Extensionality and bivalence	115
2. Composition of functions	20	5. Monics and epics by elements	123
3. Categories: first examples	23	CHAPTER 6.	
4. The pathology of abstraction	25	LOGIC CLASSICALLY CONCEIVED	125
5. Basic examples	26	1. Motivating topos logic	125
CHAPTER 3.		2. Propositions and truth-values	126
ARROWS INSTEAD OF EPSILON	37	3. The propositional calculus	129
1. Monic arrows	37	4. Boolean algebra	133
2. Epic arrows	39	5. Algebraic semantics	135
3. Iso arrows	39	6. Truth-functions as arrows	136
4. Isomorphic objects	41	7. $\mathcal{E}$ -semantics	140
5. Initial objects	43	CHAPTER 7.	
6. Terminal objects	44	ALGEBRA OF SUBOBJECTS	146
7. Duality	45	1. Complement, intersection, union	146
8. Products	46	2. Sub(d) as a lattice	151
9. Co-products	54	3. Boolean topoi	156
10. Equalisers	56	4. Internal vs. external	159
11. Limits and co-limits	58	5. Implication and its implications	162
12. Co-equalisers	60	6. Filling two gaps	166
13. The pullback	63	7. Extensionality revisited	168
14. Pushouts	68	CHAPTER 8.	
15. Completeness	69	INTUITIONISM AND ITS LOGIC	173
16. Exponentiation	70	1. Constructivist philosophy	173
CHAPTER 4.		2. Heyting's calculus	177
INTRODUCING TOPOI	75	3. Heyting algebras	178
1. Subobjects	75	4. Kripke semantics	187
2. Classifying subobjects	79		

CHAPTER 9.		CHAPTER 13.	
FUNCTORS	194	ARITHMETIC	332
1. The concept of functor	194	1. Topoi as foundations	332
2. Natural transformations	198	2. Primitive recursion	335
3. Functor categories	202	3. Peano postulates	347
CHAPTER 10.		CHAPTER 14.	
SET CONCEPTS AND VALIDITY	211	LOCAL TRUTH	359
1. Set concepts	211	1. Stacks and sheaves	359
2. Heyting algebras in $\mathbf{P}$	213	2. Classifying stacks and sheaves	368
3. The subobject classifier in $\mathbf{Set}^{\mathbf{P}}$	215	3. Grothendieck topoi	374
4. The truth arrows	221	4. Elementary sites	378
5. Validity	223	5. Geometric modality	381
6. Applications	227	6. Kripke-Joyal semantics	386
CHAPTER 11.		7. Sheaves as complete Ω -sets	388
ELEMENTARY TRUTH	230	8. Number systems as sheaves	413
1. The idea of a first-order lan- guage	230	CHAPTER 15.	
2. Formal language and seman- tics	234	ADJOINTNESS AND QUANTIFIERS	438
3. Axiomatics	237	1. Adjunctions	438
4. Models in a topos	238	2. Some adjoint situations	442
5. Substitution and soundness	249	3. The fundamental theorem	449
6. Kripke models	256	4. Quantifiers	453
7. Completeness	264	CHAPTER 16.	
8. Existence and free logic	266	LOGICAL GEOMETRY	458
9. Heyting-valued sets	274	1. Preservation and reflection	459
10. High-order logic	286	2. Geometric morphisms	463
CHAPTER 12.		3. Internal logic	483
CATEGORIAL SET THEORY	289	4. Geometric logic	493
1. Axioms of choice	290	5. Theories as sites	504
2. Natural numbers objects	301	REFERENCES	521
3. Formal set theory	305	CATALOGUE OF NOTATION	531
4. Transitive sets	313	INDEX OF DEFINITIONS	541
5. Set-objects	320		
6. Equivalence of models	328		