

Contents

PREFACE

xi

PART I ESSENTIAL CONCEPTS IN STATISTICS

1 Introducing Point and Interval Estimation

3

1.1 Point Estimation / 4

1.1.1 Bernoulli Model / 4

1.1.2 Geometric Model / 6

1.1.3 Some Remarks on Bias and Consistency / 11

1.2 Interval Estimation via Simulation / 12

1.3 Interval Estimation via the Bootstrap / 18

1.3.1 Computation and Comparison with Parametric Bootstrap / 18

1.3.2 Application to Bernoulli Model and Modification / 20

1.3.3 Double Bootstrap / 24

1.3.4 Double Bootstrap with Analytic Inner Loop / 26

1.4 Bootstrap Confidence Intervals in the Geometric Model / 31

1.5 Problems / 35

2 Goodness of Fit and Hypothesis Testing

37

2.1 Empirical Cumulative Distribution Function / 38

2.1.1 The Glivenko–Cantelli Theorem / 38

2.1.2 Proofs of the Glivenko–Cantelli Theorem / 41

- 2.1.3 Example with Continuous Data and Approximate Confidence Intervals / 45
- 2.1.4 Example with Discrete Data and Approximate Confidence Intervals / 49
- 2.2 Comparing Parametric and Nonparametric Methods / 52
- 2.3 Kolmogorov–Smirnov Distance and Hypothesis Testing / 57
 - 2.3.1 The Kolmogorov–Smirnov and Anderson–Darling Statistics / 57
 - 2.3.2 Significance and Hypothesis Testing / 59
 - 2.3.3 Small-Sample Correction / 63
- 2.4 Testing Normality with KD and AD / 65
- 2.5 Testing Normality with W^2 and U^2 / 68
- 2.6 Testing the Stable Paretian Distributional Assumption: First Attempt / 69
- 2.7 Two-Sample Kolmogorov Test / 73
- 2.8 More on (Moron?) Hypothesis Testing / 74
 - 2.8.1 Explanation / 75
 - 2.8.2 Misuse of Hypothesis Testing / 77
 - 2.8.3 Use and Misuse of p -Values / 79
- 2.9 Problems / 82

3 Likelihood

85

- 3.1 Introduction / 85
 - 3.1.1 Scalar Parameter Case / 87
 - 3.1.2 Vector Parameter Case / 92
 - 3.1.3 Robustness and the MCD Estimator / 100
 - 3.1.4 Asymptotic Properties of the Maximum Likelihood Estimator / 102
- 3.2 Cramér–Rao Lower Bound / 107
 - 3.2.1 Univariate Case / 108
 - 3.2.2 Multivariate Case / 111
- 3.3 Model Selection / 114
 - 3.3.1 Model Misspecification / 114
 - 3.3.2 The Likelihood Ratio Statistic / 117
 - 3.3.3 Use of Information Criteria / 119
- 3.4 Problems / 120

4 Numerical Optimization

123

- 4.1 Root Finding / 123
 - 4.1.1 One Parameter / 124
 - 4.1.2 Several Parameters / 131
- 4.2 Approximating the Distribution of the Maximum Likelihood Estimator / 135
- 4.3 General Numerical Likelihood Maximization / 136

4.3.1	Newton–Raphson and Quasi-Newton Methods /	137
4.3.2	Imposing Parameter Restrictions /	140
4.4	Evolutionary Algorithms /	145
4.4.1	Differential Evolution /	146
4.4.2	Covariance Matrix Adaption Evolutionary Strategy /	149
4.5	Problems /	155
5	Methods of Point Estimation	157
5.1	Univariate Mixed Normal Distribution /	157
5.1.1	Introduction /	157
5.1.2	Simulation of Univariate Mixtures /	160
5.1.3	Direct Likelihood Maximization /	161
5.1.4	Use of the EM Algorithm /	169
5.1.5	Shrinkage-Type Estimation /	174
5.1.6	Quasi-Bayesian Estimation /	176
5.1.7	Confidence Intervals /	178
5.2	Alternative Point Estimation Methodologies /	184
5.2.1	Method of Moments Estimator /	185
5.2.2	Use of Goodness-of-Fit Measures /	190
5.2.3	Quantile Least Squares /	191
5.2.4	Pearson Minimum Chi-Square /	193
5.2.5	Empirical Moment Generating Function Estimator /	195
5.2.6	Empirical Characteristic Function Estimator /	198
5.3	Comparison of Methods /	199
5.4	A Primer on Shrinkage Estimation /	200
5.5	Problems /	202
 PART II FURTHER FUNDAMENTAL CONCEPTS IN STATISTICS		
6	Q-Q Plots and Distribution Testing	209
6.1	P-P Plots and Q-Q Plots /	209
6.2	Null Bands /	211
6.2.1	Definition and Motivation /	211
6.2.2	Pointwise Null Bands via Simulation /	212
6.2.3	Asymptotic Approximation of Pointwise Null Bands /	213
6.2.4	Mapping Pointwise and Simultaneous Significance Levels /	215
6.3	Q-Q Test /	217
6.4	Further P-P and Q-Q Type Plots /	219
6.4.1	(Horizontal) Stabilized P-P Plots /	219

6.4.2	Modified S-P Plots / 220	
6.4.3	MSP Test for Normality / 224	
6.4.4	Modified Percentile (Fowlkes-MP) Plots / 228	
6.5	Further Tests for Composite Normality / 231	
6.5.1	Motivation / 232	
6.5.2	Jarque-Bera Test / 234	
6.5.3	Three Powerful (and More Recent) Normality Tests / 237	
6.5.4	Testing Goodness of Fit via Binning: Pearson's X_p^2 Test / 240	
6.6	Combining Tests and Power Envelopes / 247	
6.6.1	Combining Tests / 248	
6.6.2	Power Comparisons for Testing Composite Normality / 252	
6.6.3	Most Powerful Tests and Power Envelopes / 252	
6.7	Details of a Failed Attempt / 255	
6.8	Problems / 260	
7	Unbiased Point Estimation and Bias Reduction	269
7.1	Sufficiency / 269	
7.1.1	Introduction / 269	
7.1.2	Factorization / 272	
7.1.3	Minimal Sufficiency / 276	
7.1.4	The Rao-Blackwell Theorem / 283	
7.2	Completeness and the Uniformly Minimum Variance Unbiased Estimator / 286	
7.3	An Example with i.i.d. Geometric Data / 289	
7.4	Methods of Bias Reduction / 293	
7.4.1	The Bias-Function Approach / 293	
7.4.2	Median-Unbiased Estimation / 296	
7.4.3	Mode-Adjusted Estimator / 297	
7.4.4	The Jackknife / 302	
7.5	Problems / 305	
8	Analytic Interval Estimation	313
8.1	Definitions / 313	
8.2	Pivotal Method / 315	
8.2.1	Exact Pivots / 315	
8.2.2	Asymptotic Pivots / 318	
8.3	Intervals Associated with Normal Samples / 319	
8.3.1	Single Sample / 319	
8.3.2	Paired Sample / 320	
8.3.3	Two Independent Samples / 322	
8.3.4	Welch's Method for $\mu_1 - \mu_2$ when $\sigma_1^2 \neq \sigma_2^2$ / 323	
8.3.5	Satterthwaite's Approximation / 324	

- 8.4 Cumulative Distribution Function Inversion / 326
 - 8.4.1 Continuous Case / 326
 - 8.4.2 Discrete Case / 330
- 8.5 Application of the Nonparametric Bootstrap / 334
- 8.6 Problems / 337

PART III ADDITIONAL TOPICS

9 Inference in a Heavy-Tailed Context 341

- 9.1 Estimating the Maximally Existing Moment / 342
- 9.2 A Primer on Tail Estimation / 346
 - 9.2.1 Introduction / 346
 - 9.2.2 The Hill Estimator / 346
 - 9.2.3 Use with Stable Paretian Data / 349
- 9.3 Noncentral Student's t Estimation / 351
 - 9.3.1 Introduction / 351
 - 9.3.2 Direct Density Approximation / 352
 - 9.3.3 Quantile-Based Table Lookup Estimation / 353
 - 9.3.4 Comparison of NCT Estimators / 354
- 9.4 Asymmetric Stable Paretian Estimation / 358
 - 9.4.1 Introduction / 358
 - 9.4.2 The Hint Estimator / 359
 - 9.4.3 Maximum Likelihood Estimation / 360
 - 9.4.4 The McCulloch Estimator / 361
 - 9.4.5 The Empirical Characteristic Function Estimator / 364
 - 9.4.6 Testing for Symmetry in the Stable Model / 366
- 9.5 Testing the Stable Paretian Distribution / 368
 - 9.5.1 Test Based on the Empirical Characteristic Function / 368
 - 9.5.2 Summability Test and Modification / 371
 - 9.5.3 ALHADI: The α -Hat Discrepancy Test / 375
 - 9.5.4 Joint Test Procedure / 383
 - 9.5.5 Likelihood Ratio Tests / 384
 - 9.5.6 Size and Power of the Symmetric Stable Tests / 385
 - 9.5.7 Extension to Testing the Asymmetric Stable Paretian Case / 395

10 The Method of Indirect Inference 401

- 10.1 Introduction / 401
- 10.2 Application to the Laplace Distribution / 403
- 10.3 Application to Randomized Response / 403
 - 10.3.1 Introduction / 403
 - 10.3.2 Estimation via Indirect Inference / 406

10.4 Application to the Stable Paretian Distribution / 409

10.5 Problems / 416

A Review of Fundamental Concepts in Probability Theory 419

A.1 Combinatorics and Special Functions / 420

A.2 Basic Probability and Conditioning / 423

A.3 Univariate Random Variables / 424

A.4 Multivariate Random Variables / 427

A.5 Continuous Univariate Random Variables / 430

A.6 Conditional Random Variables / 432

A.7 Generating Functions and Inversion Formulas / 434

A.8 Value at Risk and Expected Shortfall / 437

A.9 Jacobian Transformations / 451

A.10 Sums and Other Functions / 453

A.11 Saddlepoint Approximations / 456

A.12 Order Statistics / 460

A.13 The Multivariate Normal Distribution / 462

A.14 Noncentral Distributions / 465

A.15 Inequalities and Convergence / 467

A.15.1 Inequalities for Random Variables / 467

A.15.2 Convergence of Sequences of Sets / 469

A.15.3 Convergence of Sequences of Random Variables / 473

A.16 The Stable Paretian Distribution / 483

A.17 Problems / 492

A.18 Solutions / 509

REFERENCES 537

INDEX 561