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An Exploration of Dynamical Systems and Chaos

This book is conceived as a comprehensive and detailed text-book on non-linear dynamical systems with particular emphasis on the exploration of chaotic phenomena. The self-contained introductory presentation is addressed both to those who wish to study the physics of chaotic systems and non-linear dynamics intensively as well as those who are curious to learn more about the fascinating world of chaotic phenomena. Basic concepts like Poincaré section, iterated mappings, Hamiltonian chaos and KAM theory, strange attractors, fractal dimensions, Lyapunov exponents, bifurcation theory, self-similarity and renormalisation and transitions to chaos are thoroughly explained. To facilitate comprehension, mathematical concepts and tools are introduced in short sub-sections. The text is supported by numerous computer experiments and a multitude of graphical illustrations and colour plates emphasising the geometrical and topological characteristics of the underlying dynamics.

This volume is a completely revised and enlarged second edition which comprises recently obtained research results of topical interest, and has been extended to include a new section on the basic concepts of probability theory. A completely new chapter on fully developed turbulence presents the successes of chaos theory, its limitations as well as future trends in the development of complex spatio-temporal structures.

"This book will be of valuable help for my lectures"
Hermann Haken, Stuttgart

"This text-book should not be missing in any introductory lecture
on non-linear systems and deterministic chaos"
Wolfgang Kinzel, Würzburg

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