

Assuming a background only in calculus and linear algebra, this book introduces the reader, in a technically complete way, to measure theory and probability, discrete martingales, and weak convergence. It is self-contained and rigorous with a tutorial approach that leads the reader to develop basic skills in analysis and probability.

Since the prerequisites are so few, many topics are developed in short exercises when and where they are needed. This approach was adopted in order to make the subject accessible to readers without a standard background in mathematics or statistics. A reader with a background in finance, business, or engineering should be able to acquire a technical understanding of discrete martingales in the equivalent of one semester.

While the original goal was to bring discrete martingale theory to a wide readership, it has been extended so that the book also covers the basic topics of measure theory as well as giving an introduction to the central limit theorem and weak convergence. As a result, students of pure mathematics and statistics can expect to acquire a sound introduction to basic measure theory and probability from this text with the additional benefit of an exposure to probability or analysis, respectively.

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