

*A very carefully crafted introduction to the theory and some of the applications of Gröbner bases ... contains a wealth of illustrative examples and a wide variety of useful exercises, the discussion is everywhere well-motivated, and further developments and important issues are well sign-posted ... has many solid virtues and is an ideal text for beginners in the subject ... certainly an excellent text.*

**—Bulletin of the London Mathematical Society**

As the primary tool for doing explicit computations in polynomial rings in many variables, Gröbner bases are an important component of all computer algebra systems. They are also important in computational commutative algebra and algebraic geometry. This book provides a leisurely and fairly comprehensive introduction to Gröbner bases and their applications. Adams and Loustaunau cover the following topics: the theory and construction of Gröbner bases for polynomials with coefficients in a field, applications of Gröbner bases to computational problems involving rings of polynomials in many variables, a method for computing syzygy modules and Gröbner bases in modules, and the theory of Gröbner bases for polynomials with coefficients in rings. With over 120 worked-out examples and 200 exercises, this book is aimed at advanced undergraduate and graduate students. It would be suitable as a supplement to a course in commutative algebra or as a textbook for a course in computer algebra or computational commutative algebra. This book would also be appropriate for students of computer science and engineering who have some acquaintance with modern algebra.

ISBN 978-0-8218-3804-4



9 780821 838044

GSM/3

**AMS on the Web**  
**www.ams.org**



Chapter 1. Basic Theory of Gröbner Bases	1
1.1. Introduction	1
1.2. The Linear Case	7
1.3. The One Variable Case	10
1.4. Term Orders	18
1.5. Division Algorithm	25
1.6. Gröbner Bases	32
1.7. S-Polynomials and Buchberger's Algorithm	39
1.8. Reduced Gröbner Bases	46
1.9. Summary	50
Chapter 2. Applications of Gröbner Bases	53
2.1. Elementary Applications of Gröbner Bases	53
2.2. Hilbert Nullstellensatz	61
2.3. Elimination	69
2.4. Polynomial Maps	79
2.5. Some Applications to Algebraic Geometry	90
2.6. Minimal Polynomials of Elements in Field Extensions	97
2.7. The 3-Color Problem	102
2.8. Integer Programming	105
Chapter 3. Modules and Gröbner Bases	113
3.1. Modules	113
3.2. Gröbner Bases and Syzygies	118
3.3. Improvements on Buchberger's Algorithm	124
3.4. Computation of the Syzygy Module	134
3.5. Gröbner Bases for Modules	140
3.6. Elementary Applications of Gröbner Bases for Modules	152
3.7. Syzygies for Modules	161
3.8. Applications of Syzygies	171
3.9. Computation of Hom	183



3.10. Free Resolutions	194
Chapter 4. Gröbner Bases over Rings	201
4.1. Basic Definitions	202
4.2. Computing Gröbner Bases over Rings	212
4.3. Applications of Gröbner Bases over Rings	225
4.4. A Primality Test	237
4.5. Gröbner Bases over Principal Ideal Domains	246
4.6. Primary Decomposition in $R[x]$ for $R$ a PID	259
Appendix A. Computations and Algorithms	275
Appendix B. Well-ordering and Induction	277
References	279
List of Symbols	283
Index	285