

Contents

Preface	ix
Introduction	xi
Chapter 1. Foundations	1
§1.1. Metric Spaces and Compact Sets	2
§1.2. Finite-Dimensional Banach Spaces	17
§1.3. The Dual Space	25
§1.4. Hilbert Spaces	31
§1.5. Banach Algebras	35
§1.6. The Baire Category Theorem	40
§1.7. Problems	45
Chapter 2. Principles of Functional Analysis	49
§2.1. Uniform Boundedness	50
§2.2. Open Mappings and Closed Graphs	54
§2.3. Hahn–Banach and Convexity	65
§2.4. Reflexive Banach Spaces	80
§2.5. Problems	101
Chapter 3. The Weak and Weak* Topologies	109
§3.1. Topological Vector Spaces	110
§3.2. The Banach–Alaoglu Theorem	124
§3.3. The Banach–Dieudonné Theorem	130
§3.4. The Eberlein–Šmulyan Theorem	134

§3.5. The Kreĭn–Milman Theorem	140
§3.6. Ergodic Theory	144
§3.7. Problems	153
Chapter 4. Fredholm Theory	163
§4.1. The Dual Operator	164
§4.2. Compact Operators	173
§4.3. Fredholm Operators	179
§4.4. Composition and Stability	184
§4.5. Problems	189
Chapter 5. Spectral Theory	197
§5.1. Complex Banach Spaces	198
§5.2. Spectrum	208
§5.3. Operators on Hilbert Spaces	222
§5.4. Functional Calculus for Self-Adjoint Operators	234
§5.5. Gelfand Spectrum and Normal Operators	246
§5.6. Spectral Measures	261
§5.7. Cyclic Vectors	281
§5.8. Problems	288
Chapter 6. Unbounded Operators	295
§6.1. Unbounded Operators on Banach Spaces	295
§6.2. The Dual of an Unbounded Operator	306
§6.3. Unbounded Operators on Hilbert Spaces	313
§6.4. Functional Calculus and Spectral Measures	326
§6.5. Problems	342
Chapter 7. Semigroups of Operators	349
§7.1. Strongly Continuous Semigroups	350
§7.2. The Hille–Yosida–Phillips Theorem	363
§7.3. The Dual Semigroup	377
§7.4. Analytic Semigroups	388
§7.5. Banach Space Valued Measurable Functions	404
§7.6. Inhomogeneous Equations	425
§7.7. Problems	439

Appendix A. Zorn and Tychonoff	445
§A.1. The Lemma of Zorn	445
§A.2. Tychonoff's Theorem	449
Bibliography	453
Notation	459
Index	461