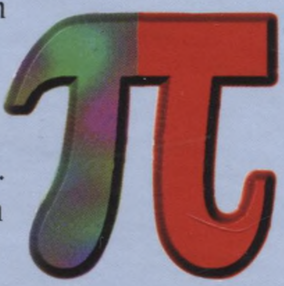
Pi : A Source Book



This book documents the history of pi from the dawn of mathematical time to the present. The story of pi reflects the most seminal, the most serious, and sometimes the most whimsical aspects of mathematics. Much significant mathematics originates with pi, and many great mathematicians have contributed to this story's unfolding.

Pi is one of the few concepts in mathematics whose mention evokes a response of recognition and interest in those not concerned professionally with the subject. Yet, despite this, no source book on pi has been published. One of the beauties of the literature on pi is that it allows for the inclusion of very modern, yet still accessible, mathematics. Mathematicians and historians of mathematics will find this book indispensable. Teachers at every level can find here ample resources for anything from individual talks and student projects to special topics courses.

The literature on pi collected herein falls into various classes. First and foremost there is a selection from the mathematical and computational literature of four millennia. There is also a variety of historical studies on the cultural significance of the number. Additionally, there is a selection of pieces that are anecdotal, fanciful, or simply amusing.

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A problem dealing with the area of a round field of given diameter.

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A conjectural explanation of how the mathematicians of ancient Egypt approximated the area of a circle.

3. Archimedes. Measurement of a Circle (~250 BC) 7
The seminal work in which Archimedes presents the first true algorithm for π.

4. Phillips. Archimedes the Numerical Analyst (1981) 15
A summary of Archimedes' work on the computation of π using modern notation.

5. Lam and Ang. Circle Measurements in Ancient China (1986) 20
This paper discusses and contains a translation of Liu Hui's (3rd century) method for evaluating π and also examines values for π given by Zu Chongzhi (429–500).

6. The Banū Mūsā: The Measurement of Plane and Solid Figures (~850) 36
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These theorems by a fifteenth century Indian mathematician give Gregory's series for arctan with remainder terms and Leibniz's series for π.

8. Hope-Jones. Ludolph (or Ludolff or Lucius) van Ceulen (1938) 51
Correspondence about van Ceulen's tombstone in reference to it containing some digits of π.

9. Viète. Variorum de Rebus Mathematicis Reponsorum Liber VII (1593) **53**
Two excerpts. One containing the first infinite expression of π, obtained by relating the area of a regular 2n-gon to that of a regular n-gon.

10. Wallis. Computation of π by Successive Interpolations (1655) **68**
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14. Roy. The Discovery of the Series Formula for π by Leibniz, Gregory, and Nilakantha (1990) **92**
A discussion of the discovery of the series $\pi/4 = 1 - 1/3 + 1/5 \ldots$

15. Jones. The First Use of π for the Circle Ratio (1706) **108**
An excerpt from Jones' book, the Synopsis Palmariorum Matheseos*: or, a* New Introduction to the Mathematics*, London, 1706.*

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The first proof of the transcendence of π.

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A summary of the action taken by the Indiana State Legislature to fix the value of π (including a copy of the actual bill that was proposed).

27. Singmaster. The Legal Values of Pi (1985) **236**
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28. Ramanujan. Squaring the Circle (1913) **240**
A geometric approximation to π.

29. Ramanujan. Modular Equations and Approximations to π (1914) **241**
Ramanujan's seminal paper on π that includes a number of striking series and algebraic approximations.

30. Watson. The Marquis and the Land Agent: A Tale of the Eighteenth Century (1933) **258**
A Presidential address to the Mathematical Association in which the author gives an account of "some of the elementary work on arcs and ellipses and other curves which led up to the idea of inverting an elliptic integral, and so laying the foundations of elliptic functions and doubly periodic functions generally."

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32. Birch. An Algorithm for Construction of Arctangent Relations (1946) **274**
The object of this note is to express π/4 as a sum of arctan relations in powers of 10.

33. Niven. A Simple Proof that π Is Irrational (1947) **276**
A very concise proof of the irrationality of π.

34. Reitwiesner. An ENIAC Determination of π and e to 2000 Decimal Places (1950) **277**
One of the first computer-based computations.

35. Schepler. The Chronology of Pi (1950) **282**
A fairly reliable outline of the history of π from 3000 BC to 1949.

36. Mahler. On the Approximation of π (1953) **306**
"The aim of this paper is to determine an explicit lower bound free of unknown constants for the distance of π from a given rational or algebraic number."

37. Wrench, Jr. The Evolution of Extended Decimal Approximations to π (1960) **319**
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38. Shanks and Wrench, Jr. Calculation of π to 100,000 Decimals (1962) **326**
A landmark computation of π to more than 100,000 places.

39. Sweeny. On the Computation of Euler's Constant (1963) **350**
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41. Adams. Asymptotic Diophantine Approximations to E (1966) **368**
An asymptotic estimate for the rational approximation to e which disproves the conjecture that e behaves like almost all numbers in this respect.

42. Mahler. Applications of Some Formulae by Hermite to the Approximations of Exponentials of Logarithms (1967) **372**
An important extension of Hilbert's approach to the study of transcendence.

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A collection of mathematical stories and anecdotes about π.

44. Eves. Mathematical Circles Revisited; A Second Collection of Mathematical Stories and Anecdotes (excerpt) (1971) **402**
A further collection of mathematical stories and anecdotes about π.

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A unifying account of some of the methods used for computing the lemniscate constants.

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The first quadratically converging algorithm for π based on Gauss's AGM and on Legendre's relation for elliptic integrals.

47. Brent. Fast Multiple-Precision Evaluation of Elementary Functions (1976) **424**
"This paper contains the 'Gauss-Legendre' method and some different algorithms for log and exp (using Landen transformations)."

48. Beukers. A Note on the Irrationality of $\zeta(2)$ and $\zeta(3)$ (1979) **434**
A short and elegant recasting of Apéry's proof of the irrationality of $\zeta(3)$ (and $\zeta(2)$).

49. van der Poorten. A Proof that Euler Missed . . . Apéry's Proof of the Irrationality of $\zeta(3)$ (1979) **439**
An illuminating account of Apéry's astonishing proof of the irrationality of $\zeta(3)$.

50. Brent and McMillan. Some New Algorithms for High-Precision Computation of Euler's Constant (1980) **448**
Several new algorithms for high precision calculation of Euler's constant, including one which was used to compute 30,100 decimal places.

51. Apostol. A Proof that Euler Missed: Evaluating $\zeta(2)$ the Easy Way (1983) **456**
This note shows that one of the double integrals considered by Beukers ([48] in the table of contents) can be used to establish directly that $\zeta(2) = \pi^2/6$.

52. O'Shaughnessy. Putting God Back in Math (1983) **458**
An article about the Institute of Pi Research, an organization that "pokes fun at creationists by pointing out that even the Bible makes mistakes."

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More connections between π and modular equations.

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An extensive study of the complex analytic properties of the AGM.

56. Borwein and Borwein. The Arithmetic–Geometric Mean and Fast Computation of Elementary Functions (1984) **537**
The relationship between the AGM iteration and fast computation of elementary functions (one of the by-products is an algorithm for π).

57. Newman. A Simplified Version of the Fast Algorithms of Brent and Salamin (1984) **553**
Elementary algorithms for evaluating e^x and π using the Gauss AGM without explicit elliptic function theory.

58. Wagon. Is Pi Normal? (1985) **557**
A discussion of the conjecture that π has randomly distributed digits.

59. Keith. Circle Digits: A Self-Referential Story (1986) **560**
A mnemonic for the first 402 decimal places of π.

60. Bailey. The Computation of π to 29,360,000 Decimal Digits Using Borweins' Quartically Convergent Algorithm (1988) **562**
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This excerpt describes "Ramanujan's original quadratic period–quasiperiod relations for elliptic curves with complex multiplication and their applications to representations of fractions of π and other logarithms in terms of rapidly convergent nearly integral (hypergeometric) series."

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66. Beukers, Bézivin, and Robba. An Alternative Proof of the Lindemann–Weierstrass Theorem (1990) **649**
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69. Keith. Pi Mnemonics and the Art of Constrained Writing (1996) 659
A mnemonic for π based on Edgar Allen Poe's poem "The Raven."

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