

Historical Prologue

Chapter 1 *The Multitype Galton–Watson Process (The Positively Regular Case)*

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