

Many important real-life problems in mathematics, physics, chemistry, biology, engineering, economics, sociology and psychology are modelled using the tools and techniques of ordinary differential equations (ODEs). This book on ODE discusses relevant topics including first and second order linear equations, initial value problems and qualitative theory. The text covers two-point boundary value problems for second order linear and nonlinear equations. Using two linearly independent solutions, a Green's function is also constructed for given boundary conditions.

The authors emphasize the use of calculus concepts in justification and analysis of equations to get solutions in explicit form. While discussing first order linear systems, tools from linear algebra are used and the importance of these tools is clearly explained. Real-life applications are interspersed throughout. Additionally readers can find the methods and tricks to solve numerous mathematical problems with sufficient derivations and explanations. The first few chapters can be used for an undergraduate course on ODE, and later chapters can be used at graduate level.

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