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This text emphasizes rigorous mathematical techniques for the analysis of boundary value problems for ODEs arising in applications. The emphasis is on proving existence of solutions, but there is also a substantial chapter on uniqueness and multiplicity questions and several chapters which deal with the asymptotic behavior of solutions with respect to either the independent variable or some parameter. These equations may give special solutions of important PDEs, such as steady state or traveling wave solutions. Often two, or even three, approaches to the same problem are described. The advantages and disadvantages of different methods are discussed.



This book gives complete classical proofs, while also emphasizing the importance of modern methods, especially when extensions to infinite dimensional settings are needed. There are some new results as well as new and improved proofs of known theorems. The final chapter presents three unsolved problems which have received much attention over the years.

Both graduate students and more experienced researchers will be interested in the power of classical methods for problems which have also been studied with more abstract techniques. The presentation should be more accessible to mathematically inclined researchers from other areas of science and engineering than most graduate texts in mathematics.

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