

The second volume of this modern account of Kaehlerian geometry and Hodge theory starts with the topology of families of algebraic varieties. Proofs of the Lefschetz theorem on hyperplane sections, the Picard–Lefschetz study of Lefschetz pencils, and Deligne theorems on the degeneration of the Leray spectral sequence and the global invariant cycles follow. The main results of the second part are the generalized Noether–Lefschetz theorems, the generic triviality of the Abel–Jacobi maps, and most importantly Nori’s connectivity theorem, which generalizes the above. The last part of the book is devoted to the relationships between Hodge theory and algebraic cycles. The book concludes with the example of cycles on abelian varieties, where some results of Bloch and Beauville, for example, are expounded. The text is complemented by exercises giving useful results in complex algebraic geometry. It will be welcomed by researchers in both algebraic and differential geometry.

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