
Contents

Preface	VII
List of Abbreviations and Symbols	XVII
List of Figures	XIX

Part I Statistical Background for Nonparametric Statistics and Functional Data

1 Introduction to Functional Nonparametric Statistics	5
1.1 What is a Functional Variable?	5
1.2 What are Functional Datasets?	6
1.3 What are Nonparametric Statistics for Functional Data	7
1.4 Some Notation	9
1.5 Scope of the Book	10
2 Some Functional Datasets and Associated Statistical Problematics	11
2.1 Functional Chemometric Data	11
2.1.1 Description of Spectrometric Data	12
2.1.2 First Study and Statistical Problems	13
2.2 Speech Recognition Data	15
2.2.1 What are Speech Recognition Data?	15
2.2.2 First Study and Problematics	15
2.3 Electricity Consumption Data	17
2.3.1 The Data	17
2.3.2 The Forecasting Problematic	18
3 What is a Well-Adapted Space for Functional Data?	21
3.1 Closeness Notions	21

3.2	Semi-Metrics as Explanatory Tool	22
3.3	What about the Curse of Dimensionality?	25
3.4	Semi-Metrics in Practice	28
3.4.1	Functional PCA: a Tool to Build Semi-Metrics	28
3.4.2	PLS: a New Way to Build Semi-Metrics	30
3.4.3	Semi-metrics Based on Derivatives	32
3.5	R and S+ Implementations	33
3.6	What About Unbalanced Functional Data?	33
3.7	Semi-Metric Space: a Well-Adapted Framework	35
4	Local Weighting of Functional Variables	37
4.1	Why Use Kernel Methods for Functional Data?	37
4.1.1	Real Case	38
4.1.2	Multivariate Case	39
4.1.3	Functional Case	41
4.2	Local Weighting and Small Ball Probabilities	42
4.3	A Few Basic Theoretical Advances	43
Part II Nonparametric Prediction from Functional Data		
5	Functional Nonparametric Prediction Methodologies	49
5.1	Introduction	49
5.2	Various Approaches to the Prediction Problem	50
5.3	Functional Nonparametric Modelling for Prediction	52
5.4	Kernel Estimators	55
6	Some Selected Asymptotics	61
6.1	Introduction	61
6.2	Almost Complete Convergence	62
6.2.1	Regression Estimation	62
6.2.2	Conditional Median Estimation	66
6.2.3	Conditional Mode Estimation	70
6.2.4	Conditional Quantile Estimation	76
6.2.5	Complements	76
6.3	Rates of Convergence	79
6.3.1	Regression Estimation	79
6.3.2	Conditional Median Estimation	80
6.3.3	Conditional Mode Estimation	87
6.3.4	Conditional Quantile Estimation	90
6.3.5	Complements	92
6.4	Discussion, Bibliography and Open Problems	93
6.4.1	Bibliography	93
6.4.2	Going Back to Finite Dimensional Setting	94
6.4.3	Some Tracks for the Future	95

7	Computational Issues	99
7.1	Computing Estimators	99
7.1.1	Prediction via Regression	100
7.1.2	Prediction via Functional Conditional Quantiles	103
7.1.3	Prediction via Functional Conditional Mode	104
7.2	Predicting Fat Content From Spectrometric Curves	105
7.2.1	Chemometric Data and the Aim of the Problem	105
7.2.2	Functional Prediction in Action	106
7.3	Conclusion	107

Part III Nonparametric Classification of Functional Data

8	Functional Nonparametric Supervised Classification	113
8.1	Introduction and Problematic	113
8.2	Method	114
8.3	Computational Issues	116
8.3.1	kNN Estimator	116
8.3.2	Automatic Selection of the kNN Parameter	117
8.3.3	Implementation: R/S+ Routines	118
8.4	Functional Nonparametric Discrimination in Action	119
8.4.1	Speech Recognition Problem	119
8.4.2	Chemometric Data	122
8.5	Asymptotic Advances	122
8.6	Additional Bibliography and Comments	123
9	Functional Nonparametric Unsupervised Classification	125
9.1	Introduction and Problematic	125
9.2	Centrality Notions for Functional Variables	127
9.2.1	Mean	127
9.2.2	Median	129
9.2.3	Mode	130
9.3	Measuring Heterogeneity	131
9.4	A General Descending Hierarchical Method	131
9.4.1	How to Build a Partitioning Heterogeneity Index?	132
9.4.2	How to Build a Partition?	132
9.4.3	Classification Algorithm	134
9.4.4	Implementation: R/S+ Routines	135
9.5	Nonparametric Unsupervised Classification in Action	135
9.6	Theoretical Advances on the Functional Mode	137
9.6.1	Hypotheses on the Distribution	138
9.7	The Kernel Functional Mode Estimator	140
9.7.1	Construction of the Estimates	140
9.7.2	Density Pseudo-Estimator: a.co. Convergence	141
9.7.3	Mode Estimator: a.co. Convergence	144

9.7.4	Comments and Bibliography	145
9.8	Conclusions	146

Part IV Nonparametric Methods for Dependent Functional Data

10	Mixing, Nonparametric and Functional Statistics	153
10.1	Mixing: a Short Introduction	153
10.2	The Finite-Dimensional Setting: a Short Overview	154
10.3	Mixing in Functional Context	155
10.4	Mixing and Nonparametric Functional Statistics	156
11	Some Selected Asymptotics	159
11.1	Introduction	159
11.2	Prediction with Kernel Regression Estimator	160
11.2.1	Introduction and Notation	160
11.2.2	Complete Convergence Properties	161
11.2.3	An Application to the Geometrically Mixing Case	163
11.2.4	An Application to the Arithmetically Mixing Case	166
11.3	Prediction with Functional Conditional Quantiles	167
11.3.1	Introduction and Notation	167
11.3.2	Complete Convergence Properties	168
11.3.3	Application to the Geometrically Mixing Case	171
11.3.4	Application to the Arithmetically Mixing Case	175
11.4	Prediction with Conditional Mode	177
11.4.1	Introduction and Notation	177
11.4.2	Complete Convergence Properties	178
11.4.3	Application to the Geometrically Mixing Case	183
11.4.4	Application to the Arithmetically Mixing Case	184
11.5	Complements on Conditional Distribution Estimation	185
11.5.1	Convergence Results	185
11.5.2	Rates of Convergence	187
11.6	Nonparametric Discrimination of Dependent Curves	189
11.6.1	Introduction and Notation	189
11.6.2	Complete Convergence Properties	190
11.7	Discussion	192
11.7.1	Bibliography	192
11.7.2	Back to Finite Dimensional Setting	192
11.7.3	Some Open Problems	193
12	Application to Continuous Time Processes Prediction	195
12.1	Time Series and Nonparametric Statistics	195
12.2	Functional Approach to Time Series Prediction	197
12.3	Computational Issues	198
12.4	Forecasting Electricity Consumption	198

12.4.1 Presentation of the Study	198
12.4.2 The Forecasted Electrical Consumption	200
12.4.3 Conclusions.....	201

Part V Conclusions

13 Small Ball Probabilities and Semi-metrics	205
13.1 Introduction	205
13.2 The Role of Small Ball Probabilities	206
13.3 Some Special Infinite Dimensional Processes	207
13.3.1 Fractal-type Processes	207
13.3.2 Exponential-type Processes	209
13.3.3 Links with Semi-metric Choice	212
13.4 Back to the One-dimensional Setting	214
13.5 Back to the Multi- (but Finite) -Dimensional Setting.....	219
13.6 The Semi-metric: a Crucial Parameter	223
14 Some Perspectives	225
Appendix: Some Probabilistic Tools.....	227
A.1 Almost Complete Convergence	228
A.2 Exponential Inequalities for Independent r.r.v.	233
A.3 Inequalities for Mixing r.r.v.	235
References	239
Index	255