Wiley Classics Library

The **Wiley Classics Library** consists of selected books that have become recognized classics in their respective fields. With these new unabridged and inexpensive editions, Wiley hopes to extend the life of these important works by making them available to future generations of mathematicians and scientists.

Currently available in the Series:

T. W. Anderson
The Statistical Analysis of Time Series

T. S. Arthanari & Yadolah Dodge
Mathematical Programming in Statistics

Emil Artin
Geometric Algebra

Norman T. J. Bailey
The Elements of Stochastic Processes with Applications to the Natural Sciences

Robert G. Bartle
The Elements of Integration and Lebesgue Measure

George E. P. Box & George C. Tiao
Bayesian Inference in Statistical Analysis

R. W. Carter
Finite Groups of Lie Type: Conjugacy Classes and Complex Characters

R. W. Carter
Simple Groups of Lie Type

William G. Cochran & Gertrude M. Cox
Experimental Designs, Second Edition

Richard Courant
Differential and Integral Calculus, Volume I

Richard Courant
Differential and Integral Calculus, Volume II

Richard Courant & D. Hilbert
Methods of Mathematical Physics, Volume I

Richard Courant & D. Hilbert
Methods of Mathematical Physics, Volume II

D. R. Cox
Planning of Experiments

Harold M. S. Coxeter
Introduction to Modern Geometry, Second Edition

Charles W. Curtis & Irving Reiner
Representation Theory of Finite Groups and Associative Algebras

Charles W. Curtis & Irving Reiner
Methods of Representation Theory with Applications to Finite Groups and Orders, Volume I

Charles W. Curtis & Irving Reiner
Methods of Representation Theory with Applications to Finite Groups and Orders, Volume II

Bruno de Finetti
Theory of Probability, Volume 1

Bruno de Finetti
Theory of Probability, Volume 2

W. Edwards Deming
Sample Design in Business Research

Amos de Shalit & Herman Feshbach
Theoretical Nuclear Physics, Volume 1—Nuclear Structure

(continued on inside back cover)

ISBN 978-0-471-05059-9
90000
9 780471 050599

CHAPTER 0 FOUNDATIONAL MATERIAL 1

1. Rudiments of Several Complex Variables

Cauchy's Formula and Applications 2
Several Variables 6
Weierstrass Theorems and Corollaries 7
Analytic Varieties 12

2. Complex Manifolds

Complex Manifolds 14
Submanifolds and Subvarieties 18
De Rham and Dolbeault Cohomology 23
Calculus on Complex Manifolds 27

3. Sheaves and Cohomology

Origins: The Mittag-Leffler Problem 34
Sheaves 35
Cohomology of Sheaves 38
The de Rham Theorem 43
The Dolbeault Theorem 45

4. Topology of Manifolds

Intersection of Cycles 49
Poincaré Duality 53
Intersection of Analytic Cycles 60

5. Vector Bundles, Connections, and Curvature

Complex and Holomorphic Vector Bundles 66
Metrics, Connections, and Curvature 71

6. Harmonic Theory on Compact Complex Manifolds

The Hodge Theorem 80
Proof of the Hodge Theorem I: Local Theory 84

Proof of the Hodge Theorem II: Global Theory 92
Applications of the Hodge Theorem 100

7. Kähler Manifolds

The Kähler Condition 106
The Hodge Identities and the Hodge Decomposition 111
The Lefschetz Decomposition 118

CHAPTER 1 COMPLEX ALGEBRAIC VARIETIES 128

1. Divisors and Line Bundles

Divisors 129
Line Bundles 132
Chern Classes of Line Bundles 139

2. Some Vanishing Theorems and Corollaries

The Kodaira Vanishing Theorem 148
The Lefschetz Theorem on Hyperplane Sections 156
Theorem B 159
The Lefschetz Theorem on (1, 1)-classes 161

3. Algebraic Varieties

Analytic and Algebraic Varieties 164
Degree of a Variety 171
Tangent Spaces to Algebraic Varieties 175

4. The Kodaira Embedding Theorem

Line Bundles and Maps to Projective Space 176
Blowing Up 182
Proof of the Kodaira Theorem 189

5. Grassmannians

Definitions 193
The Cell Decomposition 194
The Schubert Calculus 197
Universal Bundles 207
The Plücker Embedding 209

CHAPTER 2 RIEMANN SURFACES AND ALGEBRAIC CURVES 212

1. Preliminaries

Embedding Riemann Surfaces 213
The Riemann-Hurwitz Formula 216

The Genus Formula 219
Cases $g = 0, 1$ 222

2. Abel's Theorem

Abel's Theorem—First Version 224
The First Reciprocity Law and Corollaries 229
Abel's Theorem—Second Version 232
Jacobi Inversion 235

3. Linear Systems on Curves

Reciprocity Law II 240
The Riemann-Roch Formula 243
Canonical Curves 246
Special Linear Systems I 249
Hyperelliptic Curves and Riemann's Count 253
Special Linear Systems II 259

4. Plücker Formulas

Associated Curves 263
Ramification 264
The General Plücker Formulas I 268
The General Plücker Formulas II 271
Weierstrass Points 273
Plücker Formulas for Plane Curves 277

5. Correspondences

Definitions and Formulas 282
Geometry of Space Curves 290
Special Linear Systems III 298

6. Complex Tori and Abelian Varieties

The Riemann Conditions 300
Line Bundles on Complex Tori 307
Theta-Functions 317
The Group Structure on an Abelian Variety 324
Intrinsic Formulations 326

7. Curves and Their Jacobians

Preliminaries 333
Riemann's Theorem 338
Riemann's Singularity Theorem 341
Special Linear Systems IV 349
Torelli's Theorem 359

CHAPTER 3 FURTHER TECHNIQUES 364

1. Distributions and Currents

Definitions; Residue Formulas 366
Smoothing and Regularity 373
Cohomology of Currents 382

2. Applications of Currents to Complex Analysis

Currents Associated to Analytic Varieties 385
Intersection Numbers of Analytic Varieties 392
The Levi Extension and Proper Mapping Theorems 395

3. Chern Classes

Definitions 400
The Gauss Bonnet Formulas 409
Some Remarks—Not Indispensable—Concerning Chern Classes of Holomorphic Vector Bundles 416

4. Fixed-Point and Residue Formulas

The Lefschetz Fixed-Point Formula 419
The Holomorphic Lefschetz Fixed-Point Formula 422
The Bott Residue Formula 426
The General Hirzebruch-Riemann-Roch Formula 435

5. Spectral Sequences and Applications

Spectral Sequences of Filtered and Bigraded Complexes 438
Hypercohomology 445
Differentials of the Second Kind 454
The Leray Spectral Sequence 462

CHAPTER 4 SURFACES 469

1. Preliminaries

Intersection Numbers, the Adjunction Formula, and Riemann-Roch 470
Blowing Up and Down 473
The Quadric Surface 478
The Cubic Surface 480

2. Rational Maps

Rational and Birational Maps 489
Curves on an Algebraic Surface 498
The Structure of Birational Maps Between Surfaces 510

3. Rational Surfaces I

Noether's Lemma 513
Rational Ruled Surfaces 514

The General Rational Surface 520
Surfaces of Minimal Degree 522
Curves of Maximal Genus 527
Steiner Constructions 528
The Enriques-Petri Theorem 533

4. Rational Surfaces II

The Castelnuovo-Enriques Theorem 536
The Enriques Surface 541
Cubic Surfaces Revisited 545
The Intersection of Two Quadrics in $\mathbb{P}^4$ 550

5. Some Irrational Surfaces

The Albanese Map 552
Irrational Ruled Surfaces 553
A Brief Introduction to Elliptic Surfaces 564
Kodaira Number and the Classification Theorem I 572
The Classification Theorem II 582
K-3 Surfaces 590
Enriques Surfaces 594

6. Noether's Formula

Noether's Formula for Smooth Hypersurfaces 600
Blowing Up Submanifolds 602
Ordinary Singularities of Surfaces 611
Noether's Formula for General Surfaces 618
Some Examples 628
Isolated Singularities of Surfaces 636

CHAPTER 5 RESIDUES 647

1. Elementary Properties of Residues

Definition and Cohomological Interpretation 649
The Global Residue Theorem 655
The Transformation Law and Local Duality 656

2. Applications of Residues

Intersection Numbers 662
Finite Holomorphic Mappings 667
Applications to Plane Projective Geometry 670

3. Rudiments of Commutative and Homological Algebra with Applications

Commutative Algebra 678
Homological Algebra 682
The Koszul Complex and Applications 687
A Brief Tour Through Coherent Sheaves 695

4. Global Duality

Global Ext 705
Explanation of the General Global Duality Theorem 707
Global Ext and Vector Fields with Isolated Zeros 708
Global Duality and Superabundance of Points on a Surface 712
Extensions of Modules 722
Points on a Surface and Rank-Two Vector Bundles 726
Residues and Vector Bundles 729

CHAPTER 6 THE QUADRIC LINE COMPLEX 733

1. Preliminaries: Quadrics

Rank of a Quadric 734
Linear Spaces on Quadrics 735
Linear Systems of Quadrics 741
Lines on Linear Systems of Quadrics 746
The Problem of Five Conics 749

2. The Quadric Line Complex: Introduction

Geometry of the Grassmannian $G(2,4)$ 756
Line Complexes 759
The Quadric Line Complex and Associated Kummer Surface I 762
Singular Lines of the Quadric Line Complex 767
Two Configurations 773

3. Lines on the Quadric Line Complex

The Variety of Lines on the Quadric Line Complex 778
Curves on the Variety of Lines 780
Two Configurations Revisited 784
The Group Law 787

4. The Quadric Line Complex: Reprise

The Quadric Line Complex and Associated Kummer Surface II 791
Rationality of the Quadric Line Complex 796

INDEX 805