## Contents

Editor's foreword page ..... vii
Preface ..... xix
Part I Principles and elementary applications
1 Plausible reasoning ..... 3
1.1 Deductive and plausible reasoning ..... 3
1.2 Analogies with physical theories ..... 6
1.3 The thinking computer ..... 7
1.4 Introducing the robot ..... 8
1.5 Boolean algebra ..... 9
1.6 Adequate sets of operations ..... 12
1.7 The basic desiderata ..... 17
1.8 Comments ..... 19
1.8.1 Common language vs. formal logic ..... 21
1.8.2 Nitpicking ..... 23
2 The quantitative rules ..... 24
2.1 The product rule ..... 24
2.2 The sum rule ..... 30
2.3 Qualitative properties ..... 35
2.4 Numerical values ..... 37
2.5 Notation and finite-sets policy ..... 43
2.6 Comments ..... 44
2.6.1 'Subjective' vs. 'objective' ..... 44
2.6.2 Gödel's theorem ..... 45
2.6.3 Venn diagrams ..... 47
2.6.4 The 'Kolmogorov axioms' ..... 49
3 Elementary sampling theory ..... 51
3.1 Sampling without replacement ..... 52
3.2 Logic vs. propensity ..... 60
3.3 Reasoning from less precise information ..... 64
3.4 Expectations ..... 66
3.5 Other forms and extensions ..... 68
3.6 Probability as a mathematical tool ..... 68
3.7 The binomial distribution ..... 69
3.8 Sampling with replacement ..... 72
3.8.1 Digression: a sermon on reality vs. models ..... 73
3.9 Correction for correlations ..... 75
3.10 Simplification ..... 81
3.11 Comments ..... 82
3.11.1 A look ahead ..... 84
4 Elementary hypothesis testing ..... 86
$4.1 \quad$ Prior probabilities ..... 87
4.2 Testing binary hypotheses with binary data ..... 90
4.3 Nonextensibility beyond the binary case ..... 97
4.4 Multiple hypothesis testing ..... 98
4.4.1 Digression on another derivation ..... 101
4.5 Continuous probability distribution functions ..... 107
4.6 Testing an infinite number of hypotheses ..... 109
4.6.1 Historical digression ..... 112
4.7 Simple and compound (or composite) hypotheses ..... 115
4.8 Comments ..... 116
4.8.1 Etymology ..... 116
4.8.2 What have we accomplished? ..... 117
5 Queer uses for probability theory ..... 119
5.1 Extrasensory perception ..... 119
5.2 Mrs Stewart's telepathic powers ..... 120
5.2.1 Digression on the normal approximation ..... 122
5.2.2 Back to Mrs Stewart ..... 122
5.3 Converging and diverging views ..... 126
5.4 Visual perception - evolution into Bayesianity? ..... 132
5.5 The discovery of Neptune ..... 133
5.5.1 Digression on alternative hypotheses ..... 135
5.5.2 Back to Newton ..... 137
5.6 Horse racing and weather forecasting ..... 140
5.6.1 Discussion ..... 142
5.7 Paradoxes of intuition ..... 143
5.8 Bayesian jurisprudence ..... 144
5.9 Comments ..... 146
5.9.1 What is queer? ..... 148
6 Elementary parameter estimation ..... 149
6.1 Inversion of the urn distributions ..... 149
6.2 Both $N$ and $R$ unknown ..... 150
6.3 Uniform prior ..... 152
6.4 Predictive distributions ..... 154
6.5 Truncated uniform priors ..... 157
6.6 A concave prior ..... 158
6.7 The binomial monkey prior ..... 160
6.8 Metamorphosis into continuous parameter estimation ..... 163
6.9 Estimation with a binomial sampling distribution ..... 163
6.9.1 Digression on optional stopping ..... 166
6.10 Compound estimation problems ..... 167
6.11 A simple Bayesian estimate: quantitative prior information ..... 168
6.11.1 From posterior distribution function to estimate ..... 172
6.12 Effects of qualitative prior information ..... 177
6.13 Choice of a prior ..... 178
6.14 On with the calculation! ..... 179
6.15 The Jeffreys prior ..... 181
6.16 The point of it all ..... 183
6.17 Interval estimation ..... 186
6.18 Calculation of variance ..... 186
6.19 Generalization and asymptotic forms ..... 188
6.20 Rectangular sampling distribution ..... 190
6.21 Small samples ..... 192
6.22 Mathematical trickery ..... 193
6.23 Comments ..... 195
7 The central, Gaussian or normal distribution ..... 198
7.1 The gravitating phenomenon ..... 199
7.2 The Herschel-Maxwell derivation ..... 200
7.3 The Gauss derivation ..... 202
7.4 Historical importance of Gauss's result ..... 203
7.5 The Landon derivation ..... 205
7.6 Why the ubiquitous use of Gaussian distributions? ..... 207
7.7 Why the ubiquitous success? ..... 210
7.8 What estimator should we use? ..... 211
7.9 Error cancellation ..... 213
7.10 The near irrelevance of sampling frequency distributions ..... 215
7.11 The remarkable efficiency of information transfer ..... 216
7.12 Other sampling distributions ..... 218
7.13 Nuisance parameters as safety devices ..... 219
7.14 More general properties ..... 220
7.15 Convolution of Gaussians ..... 221
7.16 The central limit theorem ..... 222
7.17 Accuracy of computations ..... 224
7.18 Galton's discovery ..... 227
7.19 Population dynamics and Darwinian evolution ..... 229
7.20 Evolution of humming-birds and flowers ..... 231
7.21 Application to economics ..... 233
7.22 The great inequality of Jupiter and Saturn ..... 234
7.23 Resolution of distributions into Gaussians ..... 235
7.24 Hermite polynomial solutions ..... 236
7.25 Fourier transform relations ..... 238
7.26 There is hope after all ..... 239
7.27 Comments ..... 240
7.27.1 Terminology again ..... 240
8 Sufficiency, ancillarity, and all that ..... 243
8.1 Sufficiency ..... 243
8.2 Fisher sufficiency ..... 245
8.2.1 Examples ..... 246
8.2.2 The Blackwell-Rao theorem ..... 247
8.3 Generalized sufficiency ..... 248
8.4 Sufficiency plus nuisance parameters ..... 249
8.5 The likelihood principle ..... 250
8.6 Ancillarity ..... 253
8.7 Generalized ancillary information ..... 254
8.8 Asymptotic likelihood: Fisher information ..... 256
8.9 Combining evidence from different sources ..... 257
8.10 Pooling the data ..... 260
8.10.1 Fine-grained propositions ..... 261
8.11 Sam's broken thermometer ..... 262
8.12 Comments ..... 264
8.12.1 The fallacy of sample re-use ..... 264
8.12.2 A folk theorem ..... 266
8.12.3 Effect of prior information ..... 267
8.12.4 Clever tricks and gamesmanship ..... 267
9 Repetitive experiments: probability and frequency ..... 270
9.1 Physical experiments ..... 271
9.2 The poorly informed robot ..... 274
9.3 Induction ..... 276
9.4 Are there general inductive rules? ..... 277
9.5 Multiplicity factors ..... 280
9.6 Partition function algorithms ..... 281
9.6.1 Solution by inspection ..... 282
9.7 Entropy algorithms ..... 285
9.8 Another way of looking at it ..... 289
9.9 Entropy maximization ..... 290
9.10 Probability and frequency ..... 292
9.11 Significance tests ..... 293
9.11.1 Implied alternatives ..... 296
9.12 Comparison of psi and chi-squared ..... 300
9.13 The chi-squared test ..... 302
9.14 Generalization ..... 304
9.15 Halley's mortality table ..... 305
9.16 Comments ..... 310
9.16.1 The irrationalists ..... 310
9.16.2 Superstitions ..... 312
10 Physics of 'random experiments' ..... 314
10.1 An interesting correlation ..... 314
10.2 Historical background ..... 315
10.3 How to cheat at coin and die tossing ..... 317
10.3.1 Experimental evidence ..... 320
10.4 Bridge hands ..... 321
10.5 General random experiments ..... 324
10.6 Induction revisited ..... 326
10.7 But what about quantum theory? ..... 327
10.8 Mechanics under the clouds ..... 329
10.9 More on coins and symmetry ..... 331
10.10 Independence of tosses ..... 335
10.11 The arrogance of the uninformed ..... 338
Part II Advanced applications
11 Discrete prior probabilities: the entropy principle ..... 343
11.1 A new kind of prior information ..... 343
11.2 Minimum $\sum p_{i}^{2}$ ..... 345
11.3 Entropy: Shannon's theorem ..... 346
11.4 The Wallis derivation ..... 351
11.5 An example ..... 354
11.6 Generalization: a more rigorous proof ..... 355
11.7 Formal properties of maximum entropy distributions ..... 358
11.8 Conceptual problems - frequency correspondence ..... 365
11.9 Comments ..... 370
12 Ignorance priors and transformation groups ..... 372
12.1 What are we trying to do? ..... 372
12.2 Ignorance priors ..... 374
12.3 Continuous distributions ..... 374
12.4 Transformation groups ..... 378
12.4.1 Location and scale parameters ..... 378
12.4.2 A Poisson rate ..... 382
12.4.3 Unknown probability for success ..... 382
12.4.4 Bertrand's problem ..... 386
12.5 Comments ..... 394
13 Decision theory, historical background ..... 397
13.1 Inference vs. decision ..... 397
13.2 Daniel Bernoulli's suggestion ..... 398
13.3 The rationale of insurance ..... 400
13.4 Entropy and utility ..... 402
13.5 The honest weatherman ..... 402
13.6 Reactions to Daniel Bernoulli and Laplace ..... 404
13.7 Wald's decision theory ..... 406
13.8 Parameter estimation for minimum loss ..... 410
13.9 Reformulation of the problem ..... 412
13.10 Effect of varying loss functions ..... 415
13.11 General decision theory ..... 417
13.12 Comments ..... 418
13.12.1 'Objectivity' of decision theory ..... 418
13.12.2 Loss functions in human society ..... 421
13.12.3 A new look at the Jeffreys prior ..... 423
13.12.4 Decision theory is not fundamental ..... 423
13.12.5 Another dimension? ..... 424
14 Simple applications of decision theory ..... 426
14.1 Definitions and preliminaries ..... 426
14.2 Sufficiency and information ..... 428
14.3 Loss functions and criteria of optimum performance ..... 430
14.4 A discrete example ..... 432
14.5 How would our robot do it? ..... 437
14.6 Historical remarks ..... 438
14.6.1 The classical matched filter ..... 439
14.7 The widget problem ..... 440
14.7.1 Solution for Stage 2 ..... 443
14.7.2 Solution for Stage 3 ..... 445
14.7.3 Solution for Stage 4 ..... 449
14.8 Comments ..... 450
15 Paradoxes of probability theory ..... 451
15.1 How do paradoxes survive and grow? ..... 451
15.2 Summing a series the easy way ..... 452
15.3 Nonconglomerability ..... 453
15.4 The tumbling tetrahedra ..... 456
15.5 Solution for a finite number of tosses ..... 459
15.6 Finite vs. countable additivity ..... 464
15.7 The Borel-Kolmogorov paradox ..... 467
15.8 The marginalization paradox ..... 470
15.8.1 On to greater disasters ..... 474
15.9 Discussion ..... 478
15.9.1 The DSZ Example \#5 ..... 480
15.9.2 Summary ..... 483
15.10 A useful result after all? ..... 484
15.11 How to mass-produce paradoxes ..... 485
15.12 Comments ..... 486
16 Orthodox methods: historical background ..... 490
16.1 The early problems ..... 490
16.2 Sociology of orthodox statistics ..... 492
16.3 Ronald Fisher, Harold Jeffreys, and Jerzy Neyman ..... 493
16.4 Pre-data and post-data considerations ..... 499
16.5 The sampling distribution for an estimator ..... 500
16.6 Pro-causal and anti-causal bias ..... 503
16.7 What is real, the probability or the phenomenon? ..... 505
16.8 Comments ..... 506
16.8.1 Communication difficulties ..... 507
17 Principles and pathology of orthodox statistics ..... 509
17.1 Information loss ..... 510
17.2 Unbiased estimators ..... 511
17.3 Pathology of an unbiased estimate ..... 516
17.4 The fundamental inequality of the sampling variance ..... 518
17.5 Periodicity: the weather in Central Park ..... 520
17.5.1 The folly of pre-filtering data ..... 521
17.6 A Bayesian analysis ..... 527
17.7 The folly of randomization ..... 531
17.8 Fisher: common sense at Rothamsted ..... 532
17.8.1 The Bayesian safety device ..... 532
17.9 Missing data ..... 533
17.10 Trend and seasonality in time series ..... 534
17.10.1 Orthodox methods ..... 535
17.10.2 The Bayesian method ..... 536
17.10.3 Comparison of Bayesian and orthodox estimates ..... 540
17.10.4 An improved orthodox estimate ..... 541
17.10.5 The orthodox criterion of performance ..... 544
17.11 The general case ..... 545
17.12 Comments ..... 550
18 The $A_{p}$ distribution and rule of succession ..... 553
18.1 Memory storage for old robots ..... 553
18.2 Relevance ..... 555
18.3 A surprising consequence ..... 557
18.4 Outer and inner robots ..... 559
18.5 An application ..... 561
18.6 Laplace's rule of succession ..... 563
18.7 Jeffreys' objection ..... 566
18.8 Bass or carp? ..... 567
18.9 So where does this leave the rule? ..... 568
18.10 Generalization ..... 568
18.11 Confirmation and weight of evidence ..... 571
18.11.1 Is indifference based on knowledge or ignorance? ..... 573
18.12 Carnap's inductive methods ..... 574
18.13 Probability and frequency in exchangeable sequences ..... 576
18.14 Prediction of frequencies ..... 576
18.15 One-dimensional neutron multiplication ..... 579
18.15.1 The frequentist solution ..... 579
18.15.2 The Laplace solution ..... 581
18.16 The de Finetti theorem ..... 586
18.17 Comments ..... 588
19 Physical measurements ..... 589
19.1 Reduction of equations of condition ..... 589
19.2 Reformulation as a decision problem ..... 592
19.2.1 Sermon on Gaussian error distributions ..... 592
19.3 The underdetermined case: $K$ is singular ..... 594
19.4 The overdetermined case: $K$ can be made nonsingular ..... 595
19.5 Numerical evaluation of the result ..... 596
19.6 Accuracy of the estimates ..... 597
19.7 Comments ..... 599
19.7.1 A paradox ..... 599
20 Model comparison ..... 601
20.1 Formulation of the problem ..... 602
20.2 The fair judge and the cruel realist ..... 603
20.2.1 Parameters known in advance ..... 604
20.2.2 Parameters unknown ..... 604
20.3 But where is the idea of simplicity? ..... 605
20.4 An example: linear response models ..... 607
20.4.1 Digression: the old sermon still another time ..... 608
20.5 Comments ..... 613
20.5.1 Final causes ..... 614
21 Outliers and robustness ..... 615
21.1 The experimenter's dilemma ..... 615
21.2 Robustness ..... 617
21.3 The two-model model ..... 619
21.4 Exchangeable selection ..... 620
21.5 The general Bayesian solution ..... 622
21.6 Pure outliers ..... 624
21.7 One receding datum ..... 625
22 Introduction to communication theory ..... 627
22.1 Origins of the theory ..... 627
22.2 The noiseless channel ..... 628
22.3 The information source ..... 634
22.4 Does the English language have statistical properties? ..... 636
22.5 Optimum encoding: letter frequencies known ..... 638
22.6 Better encoding from knowledge of digram frequencies ..... 641
22.7 Relation to a stochastic model ..... 644
22.8 The noisy channel ..... 648
Appendix A Other approaches to probability theory ..... 651
A. 1 The Kolmogorov system of probability ..... 651
A. 2 The de Finetti system of probability ..... 655
A. 3 Comparative probability ..... 656
A. 4 Holdouts against universal comparability ..... 658
A. 5 Speculations about lattice theories ..... 659
Appendix B Mathematical formalities and style ..... 661
B. 1 Notation and logical hierarchy ..... 661
B. 2 Our 'cautious approach' policy ..... 662
B. 3 Willy Feller on measure theory ..... 663
B. 4 Kronecker vs. Weierstrasz ..... 665
B. 5 What is a legitimate mathematical function? ..... 666
B.5.1 Delta-functions ..... 668
B.5.2 Nondifferentiable functions ..... 668
B.5.3 Bogus nondifferentiable functions ..... 669
B. 6 Counting infinite sets? ..... 671
B. 7 The Hausdorff sphere paradox and mathematical diseases ..... 672
B. 8 What am I supposed to publish? ..... 674
B. 9 Mathematical courtesy ..... 675
Appendix C Convolutions and cumulants ..... 677
C. 1 Relation of cumulants and moments ..... 679
C. 2 Examples ..... 680
References ..... 683
Bibliography ..... 705
Author index ..... 721
Subject index ..... 724

