

# Contents

	II Practice
<b>Preface</b>	<b>xii</b>
<b>I Vectors</b>	<b>1</b>
<b>1 Vectors</b>	<b>3</b>
1.1 Vectors . . . . .	3
1.2 Vector addition . . . . .	11
1.3 Scalar-vector multiplication . . . . .	15
1.4 Inner product . . . . .	19
1.5 Complexity of vector computations . . . . .	22
Exercises . . . . .	25
<b>2 Linear functions</b>	<b>29</b>
2.1 Linear functions . . . . .	29
2.2 Taylor approximation . . . . .	35
2.3 Regression model . . . . .	38
Exercises . . . . .	42
<b>3 Norm and distance</b>	<b>45</b>
3.1 Norm . . . . .	45
3.2 Distance . . . . .	48
3.3 Standard deviation . . . . .	52
3.4 Angle . . . . .	56
3.5 Complexity . . . . .	63
Exercises . . . . .	64
<b>4 Clustering</b>	<b>69</b>
4.1 Clustering . . . . .	69
4.2 A clustering objective . . . . .	72
4.3 The $k$ -means algorithm . . . . .	74
4.4 Examples . . . . .	79
4.5 Applications . . . . .	85
Exercises . . . . .	87

# Contents

<b>5 Linear independence</b>	<b>89</b>
5.1 Linear dependence . . . . .	89
5.2 Basis . . . . .	91
5.3 Orthonormal vectors . . . . .	95
5.4 Gram–Schmidt algorithm . . . . .	97
Exercises . . . . .	103
<b>II Matrices</b>	<b>105</b>
<b>6 Matrices</b>	<b>107</b>
6.1 Matrices . . . . .	107
6.2 Zero and identity matrices . . . . .	113
6.3 Transpose, addition, and norm . . . . .	115
6.4 Matrix–vector multiplication . . . . .	118
6.5 Complexity . . . . .	122
Exercises . . . . .	124
<b>7 Matrix examples</b>	<b>129</b>
7.1 Geometric transformations . . . . .	129
7.2 Selectors . . . . .	131
7.3 Incidence matrix . . . . .	132
7.4 Convolution . . . . .	136
Exercises . . . . .	144
<b>8 Linear equations</b>	<b>147</b>
8.1 Linear and affine functions . . . . .	147
8.2 Linear function models . . . . .	150
8.3 Systems of linear equations . . . . .	152
Exercises . . . . .	159
<b>9 Linear dynamical systems</b>	<b>163</b>
9.1 Linear dynamical systems . . . . .	163
9.2 Population dynamics . . . . .	164
9.3 Epidemic dynamics . . . . .	168
9.4 Motion of a mass . . . . .	169
9.5 Supply chain dynamics . . . . .	171
Exercises . . . . .	174
<b>10 Matrix multiplication</b>	<b>177</b>
10.1 Matrix–matrix multiplication . . . . .	177
10.2 Composition of linear functions . . . . .	183
10.3 Matrix power . . . . .	186
10.4 QR factorization . . . . .	189
Exercises . . . . .	191

<b>11 Matrix inverses</b>	<b>199</b>
11.1 Left and right inverses . . . . .	199
11.2 Inverse . . . . .	202
11.3 Solving linear equations . . . . .	207
11.4 Examples . . . . .	210
11.5 Pseudo-inverse . . . . .	214
Exercises . . . . .	217
<b>III Least squares</b>	<b>223</b>
<b>12 Least squares</b>	<b>225</b>
12.1 Least squares problem . . . . .	225
12.2 Solution . . . . .	227
12.3 Solving least squares problems . . . . .	231
12.4 Examples . . . . .	234
Exercises . . . . .	239
<b>13 Least squares data fitting</b>	<b>245</b>
13.1 Least squares data fitting . . . . .	245
13.2 Validation . . . . .	260
13.3 Feature engineering . . . . .	269
Exercises . . . . .	279
<b>14 Least squares classification</b>	<b>285</b>
14.1 Classification . . . . .	285
14.2 Least squares classifier . . . . .	288
14.3 Multi-class classifiers . . . . .	297
Exercises . . . . .	305
<b>15 Multi-objective least squares</b>	<b>309</b>
15.1 Multi-objective least squares . . . . .	309
15.2 Control . . . . .	314
15.3 Estimation and inversion . . . . .	316
15.4 Regularized data fitting . . . . .	325
15.5 Complexity . . . . .	330
Exercises . . . . .	334
<b>16 Constrained least squares</b>	<b>339</b>
16.1 Constrained least squares problem . . . . .	339
16.2 Solution . . . . .	344
16.3 Solving constrained least squares problems . . . . .	347
Exercises . . . . .	352

<b>17 Constrained least squares applications</b>	<b>357</b>
17.1 Portfolio optimization . . . . .	357
17.2 Linear quadratic control . . . . .	366
17.3 Linear quadratic state estimation . . . . .	372
Exercises . . . . .	378
<b>18 Nonlinear least squares</b>	<b>381</b>
18.1 Nonlinear equations and least squares . . . . .	381
18.2 Gauss–Newton algorithm . . . . .	386
18.3 Levenberg–Marquardt algorithm . . . . .	391
18.4 Nonlinear model fitting . . . . .	399
18.5 Nonlinear least squares classification . . . . .	401
Exercises . . . . .	412
<b>19 Constrained nonlinear least squares</b>	<b>419</b>
19.1 Constrained nonlinear least squares . . . . .	419
19.2 Penalty algorithm . . . . .	421
19.3 Augmented Lagrangian algorithm . . . . .	422
19.4 Nonlinear control . . . . .	425
Exercises . . . . .	434
<b>Appendices</b>	<b>437</b>
<b>A Notation</b>	<b>439</b>
<b>B Complexity</b>	<b>441</b>
<b>C Derivatives and optimization</b>	<b>443</b>
C.1 Derivatives . . . . .	443
C.2 Optimization . . . . .	447
C.3 Lagrange multipliers . . . . .	448
<b>D Further study</b>	<b>451</b>
<b>Index</b>	<b>455</b>
1 Linear dynamical systems . . . . .	455
2 Population dynamics . . . . .	456
3 Epidemic dynamics . . . . .	457
4 Motion of a mass . . . . .	458
5 Supply chain dynamics . . . . .	459
Exercises . . . . .	460
841 . . . . .	461
10 Matrix multiplication	177
10.1 Matrix-matrix multiplication . . . . .	177
10.2 Composition of linear functions . . . . .	183
10.3 Matrix power . . . . .	186
10.4 QR factorization . . . . .	189
Exercises . . . . .	191