Contents

Preface **xi**

Preface to the First Edition xiii

Acknowledgments xv

1 Introduction and Synopsis **1**

1.1 Introduction 1

1.2 Synopsis 5

2 Preliminaries **9**

2.1 Introduction 9

2.2 Inference in Linear Models 9

2.3 Robustness Concepts 15

2.3.1 Finite-Sample Breakdown and Tail-Performance 20

2.4 Robust and Minimax Estimation of Location 25

2.5 Clippings from Probability and Asymptotic Theory 29

2.5.1 Modes of Convergence of Stochastic Elements 31

2.5.2 Basic Probability Inequalities 34

2.5.3 Some Useful Inequalities and Lemmas 36

2.5.4 Laws of Large Numbers and Related Inequalities 40

2.5.5 Central Limit Theorems 42

2.5.6 Limit Theorems Allied to Central Limit Theorems 48

2.5.7 Central Limit Theorems for Quadratic Forms 50

2.5.8 Contiguity of Probability Measures 51

2.5.9 Hájek–Inagaki–LeCam theorem and the LAN condition 52

2.5.10 Weak Convergence of Probability Measures 53
2.5.11 Some Important Gaussian Functions 57
2.5.12 Weak Invariance Principles 59
2.5.13 Empirical Distributional Processes 60
2.5.14 Weak Invariance Principle: Random Change of Time 64
2.5.15 Embedding Theorems and Strong Invariance Principles 64
2.5.16 Asymptotic Relative Efficiency: Concept and Measures 66
2.6 Problems 67

3 Robust Estimation of Location and Regression 69
3.1 Introduction 69
3.2 M-Estimators 70
3.3 L-Estimators 78
3.4 R-Estimators 91
3.5 Minimum Distance and Pitman Estimators 104
3.5.1 Minimum Distance Estimation 104
3.5.2 Pitman Estimators 106
3.5.3 Pitman-Type Estimators of Location 106
3.5.4 Bayes-Type Estimators of General Parameter 107
3.6 Differentiable Statistical Functions 108
3.7 Problems 112

4 Asymptotic Representations for L-Estimators 117
4.1 Introduction 117
4.2 Bahadur Representations for Sample Quantiles 119
4.3 L-Statistics with Smooth Scores 123
4.4 General L-Estimators 129
4.5 Statistical Functionals 130
4.6 Second-Order Asymptotic Distributional Representations 135
4.7 L-Estimation in Linear Model 142
4.8 Breakdown Point of L- and M-Estimators 152
4.9 Further Developments 155
4.10 Problems 157

5 Asymptotic Representations for M-Estimators 161
5.1 Introduction 161
5.2 M-Estimation of General Parameters 161
5.3 M-Estimation of Location: Fixed Scale 169
5.3.1 Possibly Discontinuous but Monotone ψ 173
5.3.2 Possibly Discontinuous and Nonmonotone ψ 175
5.3.3 Second-Order Distributional Representations 177
5.4 Studentized M-Estimators of Location 182
5.5 M-Estimation in Linear Model 191
5.6 Studentizing Scale Statistics 199
5.7 Hadamard Differentiability in Linear Models 202
5.8 Further Developments 205
5.9 Problems 206

6 Asymptotic Representations for R-Estimators 209
6.1 Introduction 209
6.2 Asymptotic Representations for R-Estimators of Location 210
6.3 Representations for R-Estimators in Linear Model 217
6.4 Regression Rank Scores 224
6.5 Inference Based on Regression Rank Scores 227
6.5.1 RR-Tests 229
6.5.2 RR-Estimators 230
6.5.3 Studentizing Scale Statistics and Regression Rank Scores 231
6.6 Bibliographical Notes 233
6.7 Problems 234

7 Asymptotic Interrelations of Estimators 237
7.1 Introduction 237
7.2 Estimators of location 239
7.3 Estimation in linear model 249
7.4 Approximation by One-Step Versions 252
7.5 Further developments 264
7.6 Problems 265

8 Robust Estimation: Multivariate Perspectives **267**
8.1 Introduction 267
8.2 The Notion of Multivariate Symmetry 268
8.3 Multivariate Location Estimation 271
8.4 Multivariate Regression Estimation 276
8.4.1 Normal Multivariate Linear Model 277
8.4.2 General Multivariate Linear Model 277
8.5 Affine-Equivariant Robust Estimation 279
8.5.1 Smooth Affine-Equivariant L-Estimation of $\boldsymbol{\theta}$ 281
8.5.2 Affine-Equivariant Regression Estimation 288
8.5.3 Additional Remarks and Comments 290
8.6 Efficiency and Minimum Risk Estimation 291
8.7 Stein-Rule Estimators and Minimum Risk Efficiency 296
8.7.1 Location Model 297
8.7.2 Extension to the Linear Model 301
8.8 Robust Estimation of Multivariate Scatter 306
8.9 Some Complementary and Supplementary Notes 308
8.10 Problems 310

9 Robust Tests and Confidence Sets **315**
9.1 Introduction 315
9.2 M-Tests and R-Tests 316
9.2.1 M-Tests of Location 316
9.2.2 M-Tests in Linear Model 319
9.2.3 R-Tests 322
9.2.4 Robustness of Tests 324
9.2.5 Some Remarks on the Wald-Type Tests 327
9.3 Minimax Tests 328
9.4 Robust Confidence Sets 329
9.4.1 Type I Confidence Intervals 330
9.4.2 Type II Confidence Intervals 337
9.5 Multiparameter Confidence Sets 342
9.6 Affine-Equivariant Tests and Confidence Sets 346
9.7 Problems 349

Appendix **351**

Uniform Asymptotic Linearity 351

References **357**

Subject index **385**

Author index **390**