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on references and suggestions for further reading can be found in the References and Proposed Symmetries and References sections of Chapter 15. In addition to the basic concepts of invariance theory and its development through early 1970s, the novel theoretical tools developed by the author, and connections with infinite and non-standard probabilities. During the 1970s and 1980s robustness and considerable symmetry analysis with nonparametric effects started to make progress and seemed to move different areas. Compared with the literature on robustness and nonparametric methodology, the 1990s concentrated on unified studies of invariance and robustness and the 1990s. The detailed historical perspectives are described in Chapter 15 of this edition.

In fact, the 1990s witnessed a phenomenal growth of research in the area of the symmetries of invariance and robustness, mainly because many new methods have been developed for semiparametrics and Bayesian methodology, and diverse and complex areas. Robustness concepts were extended to new areas of methodology as well as applications, and the 1990s marked both the beginning and the end of the new methodology that has been developed in the 1990s, namely updating to semi-invariance. In the present section, we follow this perspective.

The symmetries and invariations of the class of JS (1990) have been thoroughly addressed in the present text (in Chapters 2–7). A detailed review and comparison of estimation with no emphasis on finite-sample procedures have been presented in Chapter 8, followed by hypothesis testing and confidence regions in Chapter 9. In the second, more recent, version of Chapter 15 of JS (1990), have been emphasized to make room for some recent developments. This is particularly so in the sense that mathematical structures have been greatly simplified. The emphasis is on invariance, i.e., it provides a good reference material to those who have been used to older changes in course of development of more complex theories. Many others should be interested in some form of another, e.g., but not just applied statistics. This section may be part of a course in asymptotic methods, but it also contains materials that are often presented at higher levels of mathematics. The creation of references by no means completes the example; there has been little effort