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Dynamic linear models were developed in engineering in the early 1960's, to monitor and control dynamic systems, although previous results can be found in the statistical literature and go back to Tinetti (1890). Early financial applications have been in the Arnold and Polaris missile programs (see, e.g., Christensen, 1984), but in the last decades dynamic linear models and more generally state space models, have enjoyed an enormous impulse, with applications in an extremely wide range of fields, from biology to economics, from engineering and quality control to environmental studies, from geophysical science to genetics. This impressive growth of applications is largely due to the possibility of solving computational difficulties using modern Monte Carlo methods in a Bayesian framework. This book is an introduction to Bayesian modeling and forecasting of time series using dynamic linear models, presenting the basic concepts and techniques, and illustrating as it progresses the practical implementation.

Statistical time series analysis using dynamic linear models were largely developed in the 1970-80's, and state space models are nowadays a focus of interest. In fact, the reader used to time series analysis will find that ARMA models and Box-Jenkins model specification may find the state space approach a bit difficult at first, but the powerful framework offered by dynamic Bayesian models of observed data and state space models will allow these gaining a new perspective. State space models can be seen as a natural extension of ARMA models, and can be naturally regarded in terms of dynamic models. But dynamic linear models offer much more flexibility in treating non-stationary time series or changing structural parameters, and are often more easily interpretable, and the more general class of state space models extends the analysis to non-Gaussian and non-linear dynamic systems. There are, of course, different approaches to estimate dynamic linear models, via generalized likelihood methods, minimum variance unbiased estimation, or via the Bayesian approach, but several advantages both methodological and computational. Kalman (1960) already underlines some basic concepts of dynamic linear models that we would say are proper to the Bayesian approach. A first step is moving from a deterministic to a stochastic system, the uncertainty