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Spectroscopy and the quantum states of molecules

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This book is organized into four parts. Part 1 introduces geometrical symmetry and true symmetry, and discusses how point group symmetry can be approximated from true symmetry. Part 2 shows how these two types of symmetry are used in solving problems. These two parts could be a book in themselves but we felt it appropriate to add the introductory part 1 in order to give the reader with a brief account of spectroscopy, quantum mechanics and the derivation of molecular wavefunctions. In the final part 4, we develop some advanced ideas and discuss current research on symmetry; the latter focuses on experiments that are being made, using atomic and molecular spectroscopy and other methods, to determine the extent of the breakdown of each of the symmetries that are invoked in describing matter.

Throughout the text, we introduce 'shadow boxes' such as this to focus attention on a particularly significant statement.

At the end of each chapter in parts 1, 2 and 3, we have included problems involving the application of the ideas developed in the chapter. The answers to selected problems are given in appendix A at the end of the book; the problems