TABLE OF CONTENTS

1 Introduction	7
1.1 Objective of the Publication	7
1.2 Current Situation	9
1.3 Research Methodology	11
1.4 Content and Structure	11
1.5 Terminological Notes	12
2 Theoretical Foundations	15
2.1 Piaget's Theory of Cognitive Development	16
2.2 Vygotsky's Social Development Theory	20
2.3 Bruner's Theory of Representations	22
2.4 Theory of Generic Models according to Hejný	23
2.5 Van Hiele Models of Geometric Thought	28
2.5.1 Levels of Geometric Thought	29
2.5.1.1 Level 0: Visualization	29
2.5.1.2 Level 1: Analysis 2.5.1.3 Level 2: Abstraction	31
2.5.1.4 Level 3: Deduction	32 32
2.5.1.5 Level 4: Rigor	33
2.5.2 Phases of Learning	33
2.5.3 Further Properties and Reflections of the van Hiele Model	34
2.6 Implication of Paradigms of the Cognitive Process on the Research	
Conception	40
3 Research Methodology	43
3.1 Intent and Objectives of the Research	43
3.2 Research Methodology	45
3.2.1 Research sample	45
3.2.2 Means of Research	46
3.2.3 Research Design	47
3.2.4 Data Recording, Analyses and Interpretation	48
3.2.5 Summary of the Research Methodology	50

4	Geometric Thinking of Children of Pre-school Age	
	(Analysis of Results)	51
4.1	Research Methods and Tools	51
4.2	Characteristic of the Research Sample	56
4.3	Objectives and Methods of Data Analysis	56
4.4	Outcomes and Interpretation (pre-school children)	58
	4.4.1 Naming of Planar Shapes: Naming the Shape by Picture	59
	4.4.2 Identification of Planar Shapes: Model of Shape by Name	61
	4.4.3 Identification and Sorting of Planar Shapes (Quantitative	
	Qualitative Analysis)	63
	4.4.3.1 Pre-school Children's Conceptions About Squares	65
	4.4.3.2 Pre-school Children's Conceptions About Triangles	70
	4.4.3.3 Pre-school Children's Conceptions About Oblongs 4.4.3.4 Pre-school Children's Conceptions About Circles	78 81
	4.4.4 Other Factors Influencing the Ability to Distinguish Shapes	01
	(age, haptic manipulation)	84
4.5	Summary of Data Analysis: Pre-school Children	87
		Tables 1
5	Geometric Thinking of First-graders	
	(Analysis of Results)	91
5.1	Research Methods and Tools	91
5.2	Characteristic of the Reseach Sample	92
	Objectives and Methods of Data Analysis	93
	Outcomes and Interpretation (students in the first grade)	95
	5.4.1 Naming and Identification of Planar Shapes	95
	5.4.2 Identification and Sorting of Planar Shapes	
	(Quantitative-Qualitative Analysis)	99
	5.4.2.1 First-graders' conceptions about squares	99
	5.4.2.2 First-graders' conceptions about triangles	103
	5.4.2.3 First-graders' conceptions about oblongs 5.4.2.4 First-graders' conceptions about circles	107 112
	5.4.3 Creation of models of planar shapes on geoboard paper	116
		120
5.5	Summary of Data Analysis: First-graders	120
6	Comparison of Results of Pre-school Chidren	
	and Students in the First Grade	125
6.1	Names of planar shapes and their identification	125
	Classification of shapes according to difficulty	127

7 Geometric Thinking of Fourth-graders	
(Analysis of Results)	129
7.1 Research Tools	129
7.2 Characteristics of the Research Sample	131
7.3 Objectives and Methods of Data Analysis	132
7.4 Results and Interpretations (Fourth-graders)	
7.4.1 Names of planar shapes ("model of the shape => name of	134
the shape")	134
7.4.2 Identification of planar shapes ("name of the shape => model	10
of the shape")	136
7.4.2.1 Fourth-graders' conceptions about squares	137
7.4.2.2 Fourth-graders' conceptions about triangles	141
7.4.2.3 Fourth-graders' conceptions about oblongs	147
7.4.2.4 Fourth-graders' conceptions about circles	152
7.4.3 Significant elements and properties of planar geometric shapes 7.4.3.1 Significant elements and properties of triangles	158
7.4.3.2 Circle and their significant elements	158 159
7.4.3.3 Significant elements and properties of squares	161
7.4.3.4 Significant elements of polygons	162
7.4.3.5 Creation of a model of a polygon in a square grid	163
7.4.4 Oblongs in classes with alternative approach (case study,	
Czech republic)	164
7.5 Summary of the data analysis: fourth-graders	172
8 Selected Determinants of Shape Identification	177
	177
8.1 The Factor of Shape Difficulty	177
8.1.1 Difficulty of Models of Geometric Shapes	177
8.1.2 Difficulty of Non-models of Geometric Shapes	179
8.2 The Factors of Handouts and Groups	180
9 Discussion	405
	185
9.1 Reflection of Findings in Context of the Theories of Cognitive Processes	186
9.1.1 Age as a determinant of geometric thought	187
9.1.2 Language and speech as determinants of geometric thinking	189
9.1.3 The Continuity and Duration of Levels of Cognitive Process in	
Geometry	192
9.2 Reflection of the Research Results in Context of Geometric Concepts	194
9.2.1 Circles	195
9.2.2 Triangles	196

9.2.3 Squares	198
9.2.4 Oblongs	200
9.3 Reflexion of the Research Results in context of Educational Implications	202
9.4 Limits of the Research	206
Conclusion	209
Appendices	213
References	225
List of Figures and Tables	232
About the Authors	239