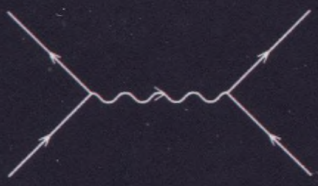

Feynman Diagram Techniques in Condensed Matter Physics

A concise introduction to Feynman diagram techniques, this book shows how they can be applied to the analysis of complex many-particle systems, and offers a review of the essential elements of quantum mechanics, solid-state physics, and statistical mechanics.

Alongside a detailed account of the method of second quantization, the book covers topics such as Green's and correlation functions, diagrammatic techniques, superconductivity, and contains several case studies. Some background knowledge in quantum mechanics, solid-state physics, and mathematical methods of physics is assumed.

Detailed derivations of formulas and in-depth examples and chapter exercises from various areas of condensed matter physics make this a valuable resource for both researchers and advanced undergraduate students in condensed-matter theory, many-body physics, and electrical engineering. Solutions to exercises are made available online.

Radi A. Jishi is a Professor of Physics at California State University. His research interests centre on condensed matter theory, carbon networks, superconductivity, and the electronic structure of crystals.

CAMBRIDGE
UNIVERSITY PRESS
www.cambridge.org

ISBN 978-1-107-65533-1

Cover designed by Hart McLeod Ltd

Preface *page* xiii

1 A brief review of quantum mechanics 1
1.1 The postulates 1
1.2 The harmonic oscillator 10
Further reading 13
Problems 13

2 Single-particle states 18
2.1 Introduction 18
2.2 Electron gas 19
2.3 Bloch states 21
2.4 Example: one-dimensional lattice 27
2.5 Wannier states 29
2.6 Two-dimensional electron gas in a magnetic field 31
Further reading 33
Problems 34

3 Second quantization 37
3.1 N-particle wave function 37
3.2 Properly symmetrized products as a basis set 38
3.3 Three examples 40
3.4 Creation and annihilation operators 42
3.5 One-body operators 47
3.6 Examples 48
3.7 Two-body operators 50
3.8 Translationally invariant system 51
3.9 Example: Coulomb interaction 52
3.10 Electrons in a periodic potential 53

3.11 Field operators 57
Further reading 61
Problems 61

4 The electron gas 65
4.1 The Hamiltonian in the jellium model 66
4.2 High density limit 69
4.3 Ground state energy 70
Further reading 76
Problems 76

5 A brief review of statistical mechanics 78
5.1 The fundamental postulate of statistical mechanics 78
5.2 Contact between statistics and thermodynamics 79
5.3 Ensembles 81
5.4 The statistical operator for a general ensemble 85
5.5 Quantum distribution functions 87
Further reading 89
Problems 89

6 Real-time Green's and correlation functions 91
6.1 A plethora of functions 92
6.2 Physical meaning of Green's functions 95
6.3 Spin-independent Hamiltonian, translational invariance 96
6.4 Spectral representation 98
6.5 Example: Green's function of a noninteracting system 106
6.6 Linear response theory 109
6.7 Noninteracting electron gas in an external potential 114
6.8 Dielectric function of a noninteracting electron gas 117
6.9 Paramagnetic susceptibility of a noninteracting electron gas 117
6.10 Equation of motion 121
6.11 Example: noninteracting electron gas 122
6.12 Example: an atom adsorbed on graphene 123
Further reading 125
Problems 126

7 Applications of real-time Green's functions 130
7.1 Single-level quantum dot 130
7.2 Quantum dot in contact with a metal: Anderson's model 133
7.3 Tunneling in solids 135
Further reading 140
Problems 140

8 Imaginary-time Green's and correlation functions 143
8.1 Imaginary-time correlation function 144
8.2 Imaginary-time Green's function 146
8.3 Significance of the imaginary-time Green's function 148
8.4 Spectral representation, relation to real-time functions 151
8.5 Example: Green's function for noninteracting particles 154
8.6 Example: Green's function for 2-DEG in a magnetic field 155
8.7 Green's function and the $\hat{U}$-operator 156
8.8 Wick's theorem 162
8.9 Case study: first-order interaction 169
8.10 Cancellation of disconnected diagrams 174
Further reading 176
Problems 176

9 Diagrammatic techniques 179
9.1 Case study: second-order perturbation in a system of fermions 179
9.2 Feynman rules in momentum-frequency space 186
9.3 An example of how to apply Feynman rules 192
9.4 Feynman rules in coordinate space 193
9.5 Self energy and Dyson's equation 196
9.6 Energy shift and the lifetime of excitations 197
9.7 Time-ordered diagrams: a case study 199
9.8 Time-ordered diagrams: Dzyaloshinski's rules 204
Further reading 210
Problems 210

10 Electron gas: a diagrammatic approach 213
10.1 Model Hamiltonian 213
10.2 The need to go beyond first-order perturbation theory 214
10.3 Second-order perturbation theory: still inadequate 216
10.4 Classification of diagrams according to the degree of divergence 218
10.5 Self energy in the random phase approximation (RPA) 219
10.6 Summation of the ring diagrams 220
10.7 Screened Coulomb interaction 222
10.8 Collective electronic density fluctuations 223
10.9 How do electrons interact? 227
10.10 Dielectric function 229
10.11 Plasmons and Landau damping 234
10.12 Case study: dielectric function of graphene 239

Further reading 244
Problems 245

11 Phonons, photons, and electrons 247
11.1 Lattice vibrations in one dimension 248
11.2 One-dimensional diatomic lattice 252
11.3 Phonons in three-dimensional crystals 254
11.4 Phonon statistics 255
11.5 Electron–phonon interaction: rigid-ion approximation 256
11.6 Electron–LO phonon interaction in polar crystals 261
11.7 Phonon Green's function 262
11.8 Free-phonon Green's function 263
11.9 Feynman rules for the electron–phonon interaction 265
11.10 Electron self energy 266
11.11 The electromagnetic field 269
11.12 Electron–photon interaction 272
11.13 Light scattering by crystals 273
11.14 Raman scattering in insulators 276
Further reading 281
Problems 281

12 Superconductivity 284
12.1 Properties of superconductors 284
12.2 The London equation 289
12.3 Effective electron–electron interaction 291
12.4 Cooper pairs 295
12.5 BCS theory of superconductivity 299
12.6 Mean field approach 304
12.7 Green's function approach to superconductivity 309
12.8 Determination of the transition temperature 316
12.9 The Nambu formalism 317
12.10 Response to a weak magnetic field 319
12.11 Infinite conductivity 325
Further reading 326
Problems 326

13 Nonequilibrium Green's function 331
13.1 Introduction 331
13.2 Schrödinger, Heisenberg, and interaction pictures 332
13.3 The malady and the remedy 336
13.4 Contour-ordered Green's function 341

13.5 Kadanoff–Baym and Keldysh contours 343
13.6 Dyson's equation 347
13.7 Langreth rules 349
13.8 Keldysh equations 351
13.9 Steady-state transport 352
13.10 Noninteracting quantum dot 360
13.11 Coulomb blockade in the Anderson model 363
Further reading 366
Problems 366

Appendix A: Second quantized form of operators 369
Appendix B: Completing the proof of Dzyaloshinski's rules 375
Appendix C: Lattice vibrations in three dimensions 378
Appendix D: Electron–phonon interaction in polar crystals 385

References 390
Index 394