



# Quantum Information Theory

Developing many of the major, exciting pre- and post-millennium developments from the ground up, this book is an ideal entry point for graduate students into quantum information theory. Significant attention is given to quantum mechanics for quantum information theory, and careful studies of the important protocols of teleportation, super-dense coding, and entanglement distribution are presented.

In this new edition, readers can expect to find over 100 pages of new material, including detailed discussions of Bell's theorem, the CHSH game, Tsirelson's theorem, the axiomatic approach to quantum channels, the definition of the diamond norm and its interpretation, and a proof of the Choi-Kraus theorem. Discussion of the importance of the quantum dynamic capacity formula has been completely revised, and many new exercises and references have been added. This new edition will be warmly welcomed by the upcoming generation of quantum information theorists and the already-established community of classical information theorists.

"For years, I have been hoping that somebody would write a book on quantum information theory that was clear, comprehensive, and up to date. This is that book. And the second edition is even better than the first."

Peter Shor, Massachusetts Institute of Technology

"Mark Wilde's *Quantum Information Theory* is a natural expositor's labor of love. Accessible to anyone comfortable with linear algebra and elementary probability theory, Wilde's book brings the reader to the forefront of research in the quantum generalization of Shannon's information theory. What had been a gaping hole in the literature has been replaced by an airy edifice, scalable with the application of reasonable effort and complete with fine vistas of the landscape below. Wilde's book has a permanent place not just on my bookshelf but on my desk."

Patrick Hayden, Stanford University, California

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<b>Part I Introduction</b>	<b>1</b>
<b>1 Concepts in Quantum Shannon Theory</b>	<b>3</b>
1.1 Overview of the Quantum Theory	7
1.2 The Emergence of Quantum Shannon Theory	11
<b>2 Classical Shannon Theory</b>	<b>26</b>
2.1 Data Compression	26
2.2 Channel Capacity	35
2.3 Summary	49
<b>Part II The Quantum Theory</b>	<b>51</b>
<b>3 The Noiseless Quantum Theory</b>	<b>53</b>
3.1 Overview	54
3.2 Quantum Bits	54
3.3 Reversible Evolution	60
3.4 Measurement	67
3.5 Composite Quantum Systems	73
3.6 Entanglement	81
3.7 Summary and Extensions to Qudit States	93
3.8 Schmidt Decomposition	98
3.9 History and Further Reading	100
<b>4 The Noisy Quantum Theory</b>	<b>101</b>
4.1 Noisy Quantum States	102
4.2 Measurement in the Noisy Quantum Theory	114
4.3 Composite Noisy Quantum Systems	117
4.4 Quantum Evolutions	129
4.5 Interpretations of Quantum Channels	141
4.6 Quantum Channels are All-Encompassing	143
4.7 Examples of Quantum Channels	154

4.8	Summary	162
4.9	History and Further Reading	163
<b>5</b>	<b>The Purified Quantum Theory</b>	164
5.1	Purification	165
5.2	Isometric Evolution	168
5.3	Coherent Quantum Instrument	179
5.4	Coherent Measurement	180
5.5	History and Further Reading	181
	<b>Part III Unit Quantum Protocols</b>	183
<b>6</b>	<b>Three Unit Quantum Protocols</b>	185
6.1	Non-Local Unit Resources	186
6.2	Protocols	188
6.3	Optimality of the Three Unit Protocols	197
6.4	Extensions for Quantum Shannon Theory	199
6.5	Three Unit Qudit Protocols	200
6.6	History and Further Reading	205
<b>7</b>	<b>Coherent Protocols</b>	207
7.1	Definition of Coherent Communication	208
7.2	Implementations of a Coherent Bit Channel	210
7.3	Coherent Dense Coding	211
7.4	Coherent Teleportation	213
7.5	Coherent Communication Identity	215
7.6	History and Further Reading	216
<b>8</b>	<b>Unit Resource Capacity Region</b>	217
8.1	The Unit Resource Achievable Region	217
8.2	The Direct Coding Theorem	221
8.3	The Converse Theorem	222
8.4	History and Further Reading	226
	<b>Part IV Tools of Quantum Shannon Theory</b>	227
<b>9</b>	<b>Distance Measures</b>	229
9.1	Trace Distance	230
9.2	Fidelity	243
9.3	Relations between Trace Distance and Fidelity	253
9.4	Gentle Measurement	257
9.5	Fidelity of a Quantum Channel	260
9.6	The Hilbert–Schmidt Distance Measure	265
9.7	History and Further Reading	266

<b>10</b>	<b>Classical Information and Entropy</b>	<b>267</b>
10.1	Entropy of a Random Variable	268
10.2	Conditional Entropy	273
10.3	Joint Entropy	274
10.4	Mutual Information	275
10.5	Relative Entropy	276
10.6	Conditional Mutual Information	278
10.7	Entropy Inequalities	279
10.8	Near Saturation of Entropy Inequalities	291
10.9	Classical Information from Quantum Systems	297
10.10	History and Further Reading	299
<b>11</b>	<b>Quantum Information and Entropy</b>	<b>300</b>
11.1	Quantum Entropy	301
11.2	Joint Quantum Entropy	306
11.3	Potential yet Unsatisfactory Definitions of Conditional Quantum Entropy	309
11.4	Conditional Quantum Entropy	311
11.5	Coherent Information	313
11.6	Quantum Mutual Information	315
11.7	Conditional Quantum Mutual Information	318
11.8	Quantum Relative Entropy	321
11.9	Quantum Entropy Inequalities	327
11.10	Continuity of Quantum Entropy	339
11.11	History and Further Reading	345
<b>12</b>	<b>Quantum Entropy Inequalities and Recoverability</b>	<b>347</b>
12.1	Recoverability Theorem	347
12.2	Schatten Norms and Complex Interpolation	348
12.3	Petz Recovery Map	353
12.4	Rényi Information Measure	355
12.5	Proof of the Recoverability Theorem	359
12.6	Refinements of Quantum Entropy Inequalities	362
12.7	History and Further Reading	366
<b>13</b>	<b>The Information of Quantum Channels</b>	<b>370</b>
13.1	Mutual Information of a Classical Channel	372
13.2	Private Information of a Wiretap Channel	379
13.3	Holevo Information of a Quantum Channel	383
13.4	Mutual Information of a Quantum Channel	389
13.5	Coherent Information of a Quantum Channel	393
13.6	Private Information of a Quantum Channel	398
13.7	Summary	404
13.8	History and Further Reading	404

<b>14</b>	<b>Classical Typicality</b>	406
14.1	An Example of Typicality	407
14.2	Weak Typicality	408
14.3	Properties of the Typical Set	410
14.4	Application: Data Compression	412
14.5	Weak Joint Typicality	414
14.6	Weak Conditional Typicality	417
14.7	Strong Typicality	420
14.8	Strong Joint Typicality	429
14.9	Strong Conditional Typicality	431
14.10	Application: Channel Capacity Theorem	437
14.11	Concluding Remarks	442
14.12	History and Further Reading	442
<b>15</b>	<b>Quantum Typicality</b>	443
15.1	The Typical Subspace	444
15.2	Conditional Quantum Typicality	454
15.3	The Method of Types for Quantum Systems	464
15.4	Concluding Remarks	466
15.5	History and Further Reading	466
<b>16</b>	<b>The Packing Lemma</b>	467
16.1	Introductory Example	468
16.2	The Setting of the Packing Lemma	468
16.3	Statement of the Packing Lemma	471
16.4	Proof of the Packing Lemma	472
16.5	Derandomization and Expurgation	478
16.6	Sequential Decoding	480
16.7	History and Further Reading	485
<b>17</b>	<b>The Covering Lemma</b>	486
17.1	Introductory Example	487
17.2	Setting and Statement of the Covering Lemma	489
17.3	Operator Chernoff Bound	491
17.4	Proof of the Covering Lemma	495
17.5	History and Further Reading	501
<b>Part V Noiseless Quantum Shannon Theory</b>		503
<b>18</b>	<b>Schumacher Compression</b>	505
18.1	The Information-Processing Task	506
18.2	The Quantum Data Compression Theorem	508
18.3	Quantum Compression Example	512

18.4	Variations on the Schumacher Theme	513
18.5	Concluding Remarks	515
18.6	History and Further Reading	515
<b>19</b>	<b>Entanglement Manipulation</b>	<b>517</b>
19.1	Sketch of Entanglement Manipulation	519
19.2	LOCC and Relative Entropy of Entanglement	522
19.3	Entanglement Manipulation Task	524
19.4	The Entanglement Manipulation Theorem	525
19.5	Concluding Remarks	535
19.6	History and Further Reading	535
<b>Part VI</b>	<b>Noisy Quantum Shannon Theory</b>	<b>537</b>
	<b>Introduction</b>	<b>539</b>
<b>20</b>	<b>Classical Communication</b>	<b>541</b>
20.1	Naive Approach: Product Measurements	543
20.2	The Information-Processing Task	546
20.3	The Classical Capacity Theorem	548
20.4	Examples of Channels	556
20.5	Superadditivity of the Holevo Information	565
20.6	Concluding Remarks	568
20.7	History and Further Reading	569
<b>21</b>	<b>Entanglement-Assisted Classical Communication</b>	<b>571</b>
21.1	The Information-Processing Task	573
21.2	A Preliminary Example	574
21.3	Entanglement-Assisted Capacity Theorem	577
21.4	The Direct Coding Theorem	578
21.5	The Converse Theorem	586
21.6	Examples of Channels	592
21.7	Concluding Remarks	597
21.8	History and Further Reading	598
<b>22</b>	<b>Coherent Communication with Noisy Resources</b>	<b>600</b>
22.1	Entanglement-Assisted Quantum Communication	601
22.2	Quantum Communication	606
22.3	Noisy Super-Dense Coding	607
22.4	State Transfer	610
22.5	Trade-off Coding	614
22.6	Concluding Remarks	622
22.7	History and Further Reading	622

<b>23</b>	<b>Private Classical Communication</b>	624
23.1	The Information-Processing Task	625
23.2	The Private Classical Capacity Theorem	627
23.3	The Direct Coding Theorem	628
23.4	The Converse Theorem	636
23.5	Discussion of Private Classical Capacity	638
23.6	History and Further Reading	641
<b>24</b>	<b>Quantum Communication</b>	642
24.1	The Information-Processing Task	643
24.2	No-Cloning and Quantum Communication	645
24.3	The Quantum Capacity Theorem	646
24.4	The Direct Coding Theorem	647
24.5	Converse Theorem	654
24.6	An Interlude with Quantum Stabilizer Codes	656
24.7	Example Channels	662
24.8	Discussion of Quantum Capacity	666
24.9	Entanglement Distillation	673
24.10	History and Further Reading	675
<b>25</b>	<b>Trading Resources for Communication</b>	679
25.1	The Information-Processing Task	680
25.2	The Quantum Dynamic Capacity Theorem	682
25.3	The Direct Coding Theorem	687
25.4	The Converse Theorem	688
25.5	Examples of Channels	699
25.6	History and Further Reading	715
<b>26</b>	<b>Summary and Outlook</b>	717
26.1	Unit Protocols	718
26.2	Noiseless Quantum Shannon Theory	718
26.3	Noisy Quantum Shannon Theory	719
26.4	Protocols Not Covered In This Book	722
26.5	Network Quantum Shannon Theory	723
26.6	Future Directions	724
<b>Appendix A Supplementary Results</b>		726
<b>Appendix B Unique Linear Extension of a Quantum Physical Evolution</b>		730
<i>References</i>		733
<i>Index</i>		754