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Series is convergent if $\operatorname{Lim}\left\{n \log \frac{n_{n}}{u_{n+1}}\right\}>1 \quad 215$

$$
\begin{equation*}
\text { Series compared with series } \Sigma a^{n} \Phi(n) \tag{216}
\end{equation*}
$$

The auxiliary series $\sum \frac{1}{n(\log n)^{p}}$
Series is convergent if $\operatorname{Lim}\left[\left\{n\left(\frac{u_{n}}{u_{n+1}}-1\right)\right\} \log n\right]>1$
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