

Introduction to **Many-Body Physics**

"Coleman begins with the basics of quantum mechanics, but manages to engagingly and deftly introduce the reader to the important ideas of many-body physics within a few chapters. [His] expert guidance then helps make a seemingly effortless step to the physics of modern correlated electron compounds. I enthusiastically recommend this book to all graduate students in physics."

Subir Sachdev, Harvard University

"This book offers the chance to learn from an accomplished expert and, just as important, from an enthusiast for quantum many-body physics. It will be essential reading for graduate students entering the rich universe of quantum many-particle phenomena [and] a chance to learn from a master. The easy-going style, the historical context and, critically, the worked examples and exercises make this book one that both novice and expert will come back to again and again."

Andrew Schofield, University of Birmingham

"This is a carefully planned and well-written book on modern physics of correlated electrons from one of the internationally recognized leaders in the field. Coleman manages to combine rigorous mathematical analysis with qualitative physics reasoning."

Andrey Chubukov, University of Wisconsin

A modern, graduate-level introduction to many-body physics in condensed matter, this textbook explains the tools and concepts needed for a research-level understanding of the correlated behavior of quantum fluids. Starting with an operator-based introduction to the quantum field theory of many-body physics, this textbook presents the Feynman diagram approach, Green's functions, and finite-temperature many-body physics before developing the path integral approach to interacting systems. Special chapters are devoted to the concepts of Landau–Fermi liquid theory, broken symmetry, conduction in disordered systems, superconductivity, local moments and the Kondo effect, and the physics of heavy-fermion metals and Kondo insulators. A strong emphasis on concepts and numerous exercises make this an invaluable course book for graduate students in condensed matter physics. It will also interest students in nuclear, atomic, and particle physics.

Piers Coleman is a Professor at the Center for Materials Theory at the Serin Physics Laboratory at Rutgers, State University of New Jersey. He invented the slave boson approach to strongly correlated electron systems and is fascinated by the emergent properties of quantum matter. Piers Coleman is also interested in science outreach and co-produced *Music of the Quantum* with his brother, the musician Jaz Coleman.

Cover illustration: "Big Bang Theory No. 2"
courtesy of Swarez Modern Art Ltd.

Cover designed by Hart McLeod Ltd

CAMBRIDGE
UNIVERSITY PRESS
www.cambridge.org

ISBN 978-0-521-86488-6



Introduction

References

1

4

1 Scales and complexity1.1 T : Time scale

5

6

1.2 L : length scale

6

1.3 N : particle number

7

1.4 C : complexity and emergence

7

References

9

2 Quantum fields

2.1 Overview

10

10

2.2 Collective quantum fields

17

2.3 Harmonic oscillator: a zero-dimensional field theory

17

2.4 Collective modes: phonons

23

2.5 The thermodynamic limit: $L \rightarrow \infty$

28

2.6 The continuum limit: $a \rightarrow 0$

31

Exercises

37

References

40

3 Conserved particles

3.1 Commutation and anticommutation algebras

42

43

3.1.1 Heuristic derivation for bosons

44

3.2 What about fermions?

46

3.3 Field operators in different bases

47

3.4 Fields as particle creation and annihilation operators

50

3.5 The vacuum and the many-body wavefunction

53

3.6 Interactions

55

3.7 Equivalence with the many-body Schrödinger equation

60

3.8 Identical conserved particles in thermal equilibrium

62

3.8.1 Generalities

62

3.8.2 Identification of the free energy: key thermodynamic properties

64

3.8.3 Independent particles

66

Exercises	67
References	69
4 Simple examples of second quantization	71
4.1 Jordan–Wigner transformation	71
4.2 The Hubbard model	78
4.3 Non-interacting particles in thermal equilibrium	80
4.3.1 Fluid of non-interacting fermions	81
4.3.2 Fluid of bosons: Bose–Einstein condensation	84
Exercises	89
References	93
5 Green’s functions	95
5.1 Interaction representation	96
5.1.1 Driven harmonic oscillator	100
5.1.2 Wick’s theorem and generating functionals	103
5.2 Green’s functions	106
5.2.1 Green’s function for free fermions	107
5.2.2 Green’s function for free bosons	110
5.3 Adiabatic concept	111
5.3.1 Gell-Mann–Low theorem	112
5.3.2 Generating function for free fermions	114
5.3.3 The spectral representation	118
5.4 Many-particle Green’s functions	121
Exercises	124
References	126
6 Landau Fermi-liquid theory	127
6.1 Introduction	127
6.2 The quasiparticle concept	129
6.3 The neutral Fermi liquid	133
6.3.1 Landau parameters	135
6.3.2 Equilibrium distribution of quasiparticles	138
6.4 Feedback effects of interactions	139
6.4.1 Renormalization of paramagnetism and compressibility by interactions	143
6.4.2 Mass renormalization	145
6.4.3 Quasiparticle scattering amplitudes	148
6.5 Collective modes	150
6.6 Charged Fermi liquids: Landau–Silin theory	153
6.7 Inelastic quasiparticle scattering	157
6.7.1 Heuristic derivation	157
6.7.2 Detailed calculation of three-body decay process	158
6.7.3 Kadowaki–Woods ratio and “local Fermi liquids”	165

6.8	Microscopic basis of Fermi-liquid theory	168
	Exercises	172
	References	174
7	Zero-temperature Feynman diagrams	176
7.1	Heuristic derivation	177
7.2	Developing the Feynman diagram expansion	183
7.2.1	Symmetry factors	189
7.2.2	Linked-cluster theorem	191
7.3	Feynman rules in momentum space	195
7.3.1	Relationship between energy and the S-matrix	197
7.4	Examples	199
7.4.1	Hartree–Fock energy	199
7.4.2	Exchange correlation	200
7.4.3	Electron in a scattering potential	202
7.5	The self-energy	206
7.5.1	Hartree–Fock self-energy	208
7.6	Response functions	210
7.6.1	Magnetic susceptibility of non-interacting electron gas	215
7.6.2	Derivation of the Lindhard function	218
7.7	The RPA (large- N) electron gas	219
7.7.1	Jellium: introducing an inert positive background	221
7.7.2	Screening and plasma oscillations	223
7.7.3	The Bardeen–Pines interaction	225
7.7.4	Zero-point energy of the RPA electron gas	228
	Exercises	229
	References	232
8	Finite-temperature many-body physics	234
8.1	Imaginary time	236
8.1.1	Representations	236
8.2	Imaginary-time Green’s functions	239
8.2.1	Periodicity and antiperiodicity	240
8.2.2	Matsubara representation	241
8.3	The contour integral method	245
8.4	Generating function and Wick’s theorem	248
8.5	Feynman diagram expansion	251
8.5.1	Feynman rules from functional derivatives	253
8.5.2	Feynman rules in frequency–momentum space	254
8.5.3	Linked-cluster theorem	258
8.6	Examples of the application of the Matsubara technique	259
8.6.1	Hartree–Fock at a finite temperature	260
8.6.2	Electron in a disordered potential	260
8.7	Interacting electrons and phonons	268

8.7.1	$\alpha^2 F$: the electron–phonon coupling function	276
8.7.2	Mass renormalization by the electron–phonon interaction	280
8.7.3	Migdal’s theorem	284
Appendix 8A	Free fermions with a source term	287
Exercises		288
References		290
9	Fluctuation–dissipation theorem and linear response theory	292
9.1	Introduction	292
9.2	Fluctuation–dissipation theorem for a classical harmonic oscillator	294
9.3	Quantum mechanical response functions	296
9.4	Fluctuations and dissipation in a quantum world	297
9.4.1	Spectral decomposition I: the correlation function $S(t - t')$	298
9.4.2	Spectral decomposition II: the retarded response function $\chi_R(t - t')$	298
9.4.3	Quantum fluctuation–dissipation theorem	300
9.4.4	Spectral decomposition III: fluctuations in imaginary time	301
9.5	Calculation of response functions	301
9.6	Spectroscopy: linking measurement and correlation	305
9.7	Electron spectroscopy	308
9.7.1	Formal properties of the electron Green’s function	308
9.7.2	Tunneling spectroscopy	310
9.7.3	ARPES, AIPES, and inverse PES	313
9.8	Spin spectroscopy	315
9.8.1	DC magnetic susceptibility	315
9.8.2	Neutron scattering	315
9.8.3	Nuclear magnetic resonance	318
9.9	Electron transport spectroscopy	321
9.9.1	Resistivity and the transport relaxation rate	321
9.9.2	Optical conductivity	324
9.9.3	The f-sum rule	326
Appendix 9A	Kramers–Kronig relation	328
Exercises		329
References		331
10	Electron transport theory	332
10.1	Introduction	332
10.2	The Kubo formula	335
10.3	Drude conductivity: diagrammatic derivation	338
10.4	Electron diffusion	343
10.5	Anderson localization	347

Exercises	354
References	356
11 Phase transitions and broken symmetry	357
11.1 Order parameter concept	357
11.2 Landau theory	359
11.2.1 Field-cooling and the development of order	359
11.2.2 The Landau free energy	361
11.2.3 Singularities at the critical point	362
11.2.4 Broken continuous symmetries: the “Mexican hat” potential	364
11.3 Ginzburg–Landau theory I: Ising order	366
11.3.1 Non-uniform solutions of Ginzburg–Landau theory	367
11.4 Ginzburg–Landau II: complex order and superflow	372
11.4.1 A “macroscopic wavefunction”	372
11.4.2 Off-diagonal long-range order and coherent states	374
11.4.3 Phase rigidity and superflow	378
11.5 Ginzburg–Landau III: charged fields	381
11.5.1 Gauge invariance	381
11.5.2 The Ginzburg–Landau equations	383
11.5.3 The Meissner effect	384
11.5.4 Vortices, flux quanta and type II superconductors	393
11.6 Dynamical effects of broken symmetry: the Anderson–Higgs mechanism	397
11.6.1 Goldstone mode in neutral superfluids	397
11.6.2 The Anderson–Higgs mechanism	399
11.6.3 Electroweak theory	402
11.7 The concept of generalized rigidity	406
11.8 Thermal fluctuations and criticality	406
11.8.1 Limits of mean-field theory: the Ginzburg criterion	410
Exercises	412
References	414
12 Path integrals	416
12.1 Coherent states and path integrals	416
12.2 Coherent states for bosons	419
12.2.1 Matrix elements and the completeness relation	420
12.3 Path integral for the partition function: bosons	424
12.3.1 Multiple bosons	427
12.3.2 Time-ordered expectation values	427
12.3.3 Gaussian path integrals	429
12.3.4 Source terms in Gaussian integrals	433
12.4 Fermions: coherent states and Grassman mathematics	435
12.4.1 Completeness and matrix elements	436
12.4.2 Path integral for the partition function: fermions	439
12.4.3 Gaussian path integral for fermions	444

12.5	The Hubbard–Stratonovich transformation	447
12.5.1	Heuristic derivation	447
12.5.2	Detailed derivation	450
12.5.3	Effective action	452
12.5.4	Generalizations to real variables and repulsive interactions	453
Appendix 12A	Derivation of key properties of bosonic coherent states	455
Appendix 12B	Grassman differentiation and integration	457
Appendix 12C	Grassman calculus: change of variables	458
Appendix 12D	Grassman calculus: Gaussian integrals	459
Exercises		460
References		462
13	Path integrals and itinerant magnetism	464
13.1	Development of the theory of itinerant magnetism	464
13.2	Path integral formulation of the Hubbard model	466
13.3	Saddle points and the mean-field theory of magnetism	469
13.4	Quantum fluctuations in the magnetization	477
Exercises		482
References		484
14	Superconductivity and BCS theory	486
14.1	Introduction: early history	486
14.2	The Cooper instability	490
14.3	The BCS Hamiltonian	496
14.3.1	Mean-field description of the condensate	498
14.4	Physical picture of BCS theory: pairs as spins	499
14.4.1	Nambu spinors	500
14.4.2	Anderson’s domain-wall interpretation of BCS theory	502
14.4.3	The BCS ground state	505
14.5	Quasiparticle excitations in BCS theory	506
14.6	Path integral formulation	511
14.6.1	Mean-field theory as a saddle point of the path integral	512
14.6.2	Computing Δ and T_c	517
14.7	The Nambu–Gor’kov Green’s function	518
14.7.1	Tunneling density of states and coherence factors	523
14.8	Twisting the phase: the superfluid stiffness	531
14.8.1	Implications of gauge invariance	532
14.8.2	Calculating the phase stiffness	535
Exercises		538
References		540
15	Retardation and anisotropic pairing	542
15.1	BCS theory with momentum-dependent coupling	542
15.2	Retardation and the Coulomb pseudopotential	545

15.3	Anisotropic pairing	548
15.4	d-wave pairing in two-dimensions	553
15.5	Superfluid ^3He	565
15.5.1	Early history: theorists predict a new superfluid	565
15.5.2	Formulation of a model	567
15.5.3	Gap equation	568
	Exercises	578
	References	580
16	Local moments and the Kondo effect	582
16.1	Strongly correlated electrons	582
16.2	Local moments	584
16.3	Asymptotic freedom in a cryostat: a brief history of the theory of magnetic moments	585
16.4	Anderson's model of local moment formation	587
16.4.1	The atomic limit	588
16.4.2	Virtual bound state formation: the non-interacting resonance	591
16.4.3	The Friedel sum rule	594
16.4.4	Mean-field theory	597
16.5	The Coulomb blockade: local moments in quantum dots	601
16.6	The Kondo effect	603
16.6.1	Adiabaticity and the Kondo resonance	606
16.7	Renormalization concept	609
16.8	Schrieffer–Wolff transformation	613
16.9	“Poor man's” scaling	619
16.9.1	Kondo calculus: Abrikosov pseudo-fermions and the Popov–Fedatov method	624
16.9.2	Universality and the resistance minimum	629
16.10	Nozières Fermi-liquid theory	634
16.10.1	Strong-coupling expansion	634
16.10.2	Phase-shift formulation of the local Fermi liquid	636
16.10.3	Experimental observation of the Kondo effect	640
16.11	Multi-channel Kondo physics	641
Appendix 16A	Derivation of Kondo integral	645
	Exercises	647
	References	651
17	Heavy electrons	656
17.1	The Kondo lattice and the Doniach phase diagram	656
17.2	The Coqblin–Schrieffer model	663
17.2.1	Construction of the model	663
17.2.2	Enhancement of the Kondo temperature	666
17.3	Large- N expansion for the Kondo lattice	668
17.3.1	Preliminaries	668

17.4	The Read–Newns path integral	670
17.4.1	The effective action	674
17.5	Mean-field theory of the Kondo impurity	679
17.5.1	The impurity effective action	679
17.5.2	Minimization of free energy	683
17.6	Mean-field theory of the Kondo lattice	685
17.6.1	Diagonalization of the Hamiltonian	685
17.6.2	Mean-field free energy and saddle point	688
17.6.3	Kondo lattice Green’s function	691
17.7	Kondo insulators	693
17.7.1	Strong-coupling expansion	693
17.7.2	Large- N treatment of the Kondo insulator	695
17.8	The composite nature of the f -electron	697
17.8.1	A thought experiment: a Kondo lattice of nuclear spins	697
17.8.2	Cooper pair analogy	698
17.9	Tunneling into heavy-electron fluids	703
17.9.1	The cotunneling Hamiltonian	704
17.9.2	Tunneling conductance and the “Fano lattice”	705
17.10	Optical conductivity of heavy electrons	709
17.10.1	Heuristic discussion	709
17.10.2	Calculation of the optical conductivity, including interband term	710
17.11	Summary	714
	Exercises	714
	References	715
18	Mixed valence, fluctuations, and topology	720
18.1	The slave boson and mixed valence	721
18.2	Path integrals with slave bosons	723
18.2.1	The link between the Kondo and Anderson lattices	727
18.3	Fluctuations about the large- N limit	733
18.3.1	Effective action and Gaussian fluctuations	734
18.3.2	Fermi liquid interactions	739
18.4	Power-law correlations, Elitzur’s theorem, and the X-ray catastrophe	740
18.5	The spectrum of spin and valence fluctuations	744
18.6	Gauge invariance and the charge of the f -electron	747
18.7	Topological Kondo insulators	751
18.7.1	The rise of topology	752
18.7.2	The Z_2 index	752
18.7.3	Topology: solution to the mystery of SmB_6	753
18.7.4	The Shockley chain	757
18.7.5	Two dimensions: the spin Hall effect	761
18.7.6	Three dimensions and SmB_6	772
18.8	Summary	773

Appendix 18A Fluctuation susceptibilities in the Kondo and infinite U

Anderson models

774

Appendix 18B Elitzur's theorem

778

Exercises

780

References

782

Epilogue: the challenge of the future

787

References

789

Author Index

790

Subject Index

792