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INTRODUCTORY REMARKS AND OUTLINE

The main tasks in statistics are the construction or choice of a statistical model for a given set of data, and the assessment and charting of statistical information in model and data.

This book is concerned with certain questions of statistical information thought to be of interest for purposes of scientific inference. It also contains an account of the theory of exponential families of probability measures, with particular reference to those questions. Besides exponential families, the most important type of statistical models are the group families, i.e. families of probability measures generated by a unitary group of transformations on the sample space. However, only the most basic facts on group families will be referred to. (Some further introductory remarks on these two types of models are given in Section 1.3.) Another limitation is that asymptotic problems are not discussed, except for a few remarks.

The reader is supposed to have a fairly broad, basic knowledge of statistical inference, and in particular to be familiar with the more conceptual aspects of likelihood and plausibility, such as are discussed in Birnbaum (1969) and Bernardo-Nielsen (1976b), respectively.

Probability functions, likelihood functions, and plausibility functions are charts of different types of statistical information. They are the three prominent instances of the concept of ods functions, due to Barnard (1949). An ods function is a real function on the space of possible experimental outcomes or on the space of hypotheses, which expresses the relative 'credibility' of the points of the space in question. It is often convenient to work with the logarithms of such functions and these are termed lods functions. For the objectives of this treatise the interest in lods (or ods) functions lies mainly in the very concept which is instrumental in bringing to the fore the duality between the sample aspect and the parameter aspect of statistical models, and in constructing prediction functions. Thus, although the concept of lods function will be referred to at a number of places, the theoretical developments relating to lods functions and presented in Barnard (1949) are not of direct relevance in the present context and will only be indicated briefly (in Section 3.1).