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$$f_{\text{m}}(v) = \rho \left[\frac{m}{2\pi kT} \right]^{1/2} \exp \left(-\frac{mv^2}{2kT} \right), \quad v \in \mathbb{R}^3, \quad (1.2)$$

where ρ , V, T are the density (number of molecules per unit volume), the stream velocity and the absolute temperature of the gas, m is the mass of a molecule and k is Boltzmann's constant. In 1872 he established the equation

$$\frac{\partial}{\partial t} f(t, x, v) + (v, \nabla_x) f(t, x, v) = \quad (1.3)$$

$$\int_{-\infty}^{\infty} dw \int_0^{\infty} r dr \int_0^{2\pi} d\phi [v - w] [f(t, x, v') f(t, x, w') - f(t, x, v) f(t, x, w)]$$