

Contents	
3.5.4 Diagonal dispersion matrix, \mathbf{D}_0 , multivariate-U	225
3.5.5 Equal diagonal entries with identity variance	225
3.5.6 Equal correlations and nullity, A_0 equalities	2
3.5.7 Simultaneous confidence intervals (Section 3.5), 96	
Large sample incorporate prior information	3
3.5.8 More general covariances, Σ_0 , diagonal Σ_0^*	11
3.6 Competing F-g Likelihoods and comparisons	102
3.6.1 Test for equal dispersion matrices	225
a. Likelihood ratio test 103 to 107	2
b. Uniform-B approximation 107 to 110	4
c. Robustness of tests, 108	
NOTATION	xix

Contents

1	PRELIMINARIES	1
1.1	Notation	1
1.2	What Is Multivariate Analysis?	3
1.3	Expectation and Covariance Operators	5
1.4	Sample Data	8
1.5	Mahalanobis Distances and Angles	10
1.6	Simultaneous Inference	11
1.6.1	<i>Simultaneous tests</i> , 11	
1.6.2	<i>Union–intersection principle</i> , 13	
1.7	Likelihood Ratio Tests	14
	Exercises 1, 14	
2	MULTIVARIATE DISTRIBUTIONS	17
2.1	Introduction	17
2.2	Multivariate Normal Distribution	17
2.3	Wishart Distribution	20
2.3.1	<i>Definition and properties</i> , 20	
2.3.2	<i>Generalized quadratics</i> , 22	
2.3.3	<i>Noncentral Wishart distribution</i> , 26	
2.3.4	<i>Eigenvalues of a Wishart matrix</i> , 27	
2.3.5	<i>Determinant of a Wishart matrix</i> , 27	
2.4	Hotelling's T^2 Distribution	28
2.4.1	<i>Central distribution</i> , 28	
2.4.2	<i>Noncentral distribution</i> , 32	
2.5.	Multivariate Beta Distributions	32
2.5.1	<i>Derivation</i> , 32	
2.5.2	<i>Multivariate beta eigenvalues</i> , 35	
2.5.3	<i>Two trace statistics</i> , 38	
a.	<i>Lawley–Hotelling statistic</i> , 38	
b.	<i>Pillai's trace statistic</i> , 39	

2.5.4	<i>U-Distribution</i> , 40	
2.5.5	<i>Summary of special distributions</i> , 42	
a.	<i>Hotelling's T^2</i> , 42	
b.	<i>U-statistic</i> , 43	
c.	<i>Maximum root statistic</i> , 43	
d.	<i>Trace statistics</i> , 43	
e.	<i>Equivalence of statistics when $m_H = 1$</i> , 43	
2.5.6	<i>Factorizations of U</i> , 45	
a.	<i>Product of beta variables</i> , 45	
b.	<i>Product of two U-statistics</i> , 48	
2.6	Rao's Distribution	50
2.7	Multivariate Skewness and Kurtosis	54
	Exercises 2, 55	
3	INFERENCE FOR THE MULTIVARIATE NORMAL	59
3.1	Introduction	59
3.2	Estimation	59
3.2.1	<i>Maximum likelihood estimation</i> , 59	
3.2.2	<i>Distribution theory</i> , 63	
3.3	Testing for the Mean	63
3.3.1	<i>Hotelling's T^2 test</i> , 63	
3.3.2	<i>Power of the test</i> , 69	
3.3.3	<i>Robustness of the test</i> , 69	
3.3.4	<i>Step-down test procedure</i> , 70	
3.4	Linear Constraints on the Mean	71
3.4.1	<i>Generalization of the paired comparison test</i> , 71	
3.4.2	<i>Some examples</i> , 72	
a.	<i>Repeated-measurement designs</i> , 72	
b.	<i>Testing specified contrasts</i> , 76	
c.	<i>Test for symmetry</i> , 76	
d.	<i>Testing for a polynomial growth trend</i> , 77	
e.	<i>Independent estimate of Σ</i> , 77	
3.4.3	<i>Minimization technique for the test statistic</i> , 77	
3.4.4	<i>Confidence intervals</i> , 81	
3.4.5	<i>Functional relationships (errors in variables regression)</i> , 85	
3.5	Inference for the Dispersion Matrix	86
3.5.1	<i>Introduction</i> , 86	
3.5.2	<i>Blockwise independence: Two blocks</i> , 87	
a.	<i>Likelihood ratio test</i> , 88	
b.	<i>Maximum root test</i> , 89	
3.5.3	<i>Blockwise independence: b Blocks</i> , 90	

3.5.4	<i>Diagonal dispersion matrix</i> , 92
3.5.5	<i>Equal diagonal blocks</i> , 94
3.5.6	<i>Equal correlations and equal variances</i> , 95
3.5.7	<i>Simultaneous confidence intervals for correlations</i> , 96
3.5.8	<i>Large-sample inferences</i> , 98
3.5.9	<i>More general covariance structures</i> , 102
3.6	Comparing Two Normal Populations 102
3.6.1	<i>Tests for equal dispersion matrices</i> , 102
a.	Likelihood ratio test, 103
b.	Union–intersection test, 105
c.	Robustness of tests, 106
d.	Robust large-sample tests, 107
3.6.2	<i>Test for equal means assuming equal dispersion matrices</i> , 108
a.	Hotelling's T^2 test, 108
b.	Effect of unequal dispersion matrices, 111
c.	Effect of nonnormality, 112
3.6.3	<i>Test for equal means assuming unequal dispersion matrices</i> , 114
3.6.4	<i>Profile analysis: Two populations</i> , 117

Exercises 3, 124

4	GRAPHICAL AND DATA - ORIENTED TECHNIQUES 127
4.1	Multivariate Graphical Displays 127
4.2	Transforming to Normality 138
4.2.1	<i>Univariate transformations</i> , 138
4.2.2	<i>Multivariate transformations</i> , 140
4.3	Distributional Tests and Plots 141
4.3.1	<i>Investigating marginal distributions</i> , 141
4.3.2	<i>Tests and plots for multivariate normality</i> , 148
4.4	Robust Estimation 156
4.4.1	<i>Why robust estimates?</i> , 156
4.4.2	<i>Estimation of location</i> , 156
a.	Univariate methods, 156
b.	Multivariate methods, 162
4.4.3	<i>Estimation of dispersion and covariance</i> , 162
a.	Univariate methods, 162
b.	Multivariate methods, 165
4.5	Outlying Observations 169

Exercises 4, 173

5 DIMENSION REDUCTION AND ORDINATION	175
5.1 Introduction	175
5.2 Principal Components	176
5.2.1 <i>Definition</i> , 176	
5.2.2 <i>Dimension reduction properties</i> , 176	
5.2.3 <i>Further properties</i> , 181	
5.2.4 <i>Sample principal components</i> , 184	
5.2.5 <i>Inference for sample components</i> , 197	
5.2.6 <i>Preliminary selection of variables</i> , 200	
5.2.7 <i>General applications</i> , 200	
5.2.8 <i>Generalized principal components analysis</i> , 203	
5.3 Biplots and <i>h</i> -Plots	204
5.4 Factor Analysis	213
5.4.1 <i>Underlying model</i> , 213	
5.4.2 <i>Estimation procedures</i> , 216	
a. <i>Method of maximum likelihood</i> , 216	
b. <i>Principal factor analysis</i> , 219	
c. <i>Estimating factor scores</i> , 220	
5.4.3 <i>Some difficulties</i> , 222	
a. <i>Principal factor analysis</i> , 225	
b. <i>Maximum likelihood factor analysis</i> , 230	
c. <i>Conclusions</i> , 231	
5.5 Multidimensional Scaling	235
5.5.1 <i>Classical (metric) solution</i> , 235	
5.5.2 <i>Nonmetric scaling</i> , 241	
5.6 Procrustes Analysis (Matching Configurations)	253
5.7 Canonical Correlations and Variates	256
5.7.1 <i>Population correlations</i> , 256	
5.7.2 <i>Sample canonical correlations</i> , 260	
5.7.3 <i>Inference</i> , 264	
5.7.4 <i>More than two sets of variables</i> , 267	
5.8 Discriminant Coordinates	269
5.9 Assessing Two-Dimensional Representations	273
5.10 A Brief Comparison of Methods	274
Exercises 5, 275	
6 DISCRIMINANT ANALYSIS	279
6.1 Introduction	279
6.2 Two Groups: Known Distributions	280
6.2.1 <i>Misclassification errors</i> , 280	
6.2.2 <i>Some allocation principles</i> , 281	
a. <i>Minimize total probability of misclassification</i> , 281	

b.	Likelihood ratio method, 285	
c.	Minimize total cost of misclassification, 285	
d.	Maximize the posterior probability, 285	
e.	Minimax allocation, 286	
f.	Summary, 287	
6.3	Two Groups: Known Distributions With Unknown Parameters	287
6.3.1	<i>General methods</i> , 287	
6.3.2	<i>Normal populations</i> , 293	
a.	Linear discriminant function, 293	
b.	Quadratic discriminant function, 297	
c.	Robustness of LDF and QDF, 297	
d.	Missing values, 300	
e.	Predictive discriminant, 301	
6.3.3	<i>Multivariate discrete distributions</i> , 303	
a.	Independent binary variables, 303	
b.	Correlated binary variables, 304	
c.	Correlated discrete variables, 306	
6.3.4	<i>Multivariate discrete-continuous distributions</i> , 306	
6.4	Two Groups: Logistic Discriminant	308
6.4.1	<i>General model</i> , 308	
6.4.2	<i>Sampling designs</i> , 309	
a.	Conditional sampling, 309	
b.	Mixture sampling, 310	
c.	Separate sampling, 310	
6.4.3	<i>Computations</i> , 312	
6.4.4	<i>LGD versus LDF</i> , 317	
6.4.5	<i>Predictive logistic model</i> , 318	
6.4.6	<i>Quadratic discrimination</i> , 319	
6.5	Two Groups: Unknown Distributions	320
6.5.1	<i>Kernel method</i> , 320	
a.	Continuous data, 320	
b.	Binary data, 322	
c.	Continuous and discrete data, 323	
6.5.2	<i>Other nonparametric methods</i> , 323	
a.	Nearest neighbor techniques, 323	
b.	Partitioning methods, 324	
c.	Distance methods, 324	
d.	Rank procedures, 324	
e.	Sequential discrimination on the variables, 326	
6.6	Utilizing Unclassified Observations	327
6.7	All Observations Unclassified (Method of Mixtures)	328
6.8	Cases of Doubt	329
6.9	More Than Two Groups	330
6.10	Selection of Variables	337

6.10.1 <i>Two groups</i> , 337	173
6.10.2 <i>More than two groups</i> , 341	
6.11 Some Conclusions	342
Exercises 6, 343	
7 CLUSTER ANALYSIS	347
7.1 Introduction	347
7.2 Proximity data	351
7.2.1 <i>Dissimilarities</i> , 351	
7.2.2 <i>Similarities</i> , 356	
7.3 Hierarchical Clustering: Agglomerative Techniques	359
7.3.1 <i>Some commonly used methods</i> , 360	
a. Single linkage (nearest neighbor) method, 360	
b. Complete linkage (farthest neighbor) method, 361	
c. Centroid method, 362	
d. Incremental sum of squares method, 363	
e. Median method, 363	
f. Group average method, 363	
g. Lance and Williams flexible method, 364	
h. Information measures, 365	
i. Other methods, 366	
7.3.2 <i>Comparison of methods</i> , 368	
a. Monotonicity, 368	
b. Spatial properties, 373	
c. Computational effort, 375	
7.4 Hierarchical Clustering: Divisive Techniques	376
7.4.1 <i>Monothetic methods</i> , 377	
7.4.2 <i>Polythetic methods</i> , 378	
7.5 Partitioning Methods	379
7.5.1 <i>Number of partitions</i> , 379	
7.5.2 <i>Starting the process</i> , 379	
7.5.3 <i>Reassigning the objects</i> , 380	
a. Minimize trace W , 382	
b. Minimize $ W $, 382	
c. Maximize trace BW^{-1} , 383	
7.5.4 <i>Other techniques</i> , 386	
7.6 Overlapping Clusters (Clumping)	387
7.7 Choosing the Number of Clusters	388
7.8 General Comments	390
Exercises 7, 392	
8 MULTIVARIATE LINEAR MODELS	395
8.1 Least Squares Estimation	395
8.2 Properties of Least Squares Estimates	399

8.3	Least Squares With Linear Constraints	403
8.4	Distribution Theory	405
8.5	Analysis of Residuals	408
8.6	Hypothesis Testing	409
8.6.1	<i>Extension of univariate theory</i> , 409	
8.6.2	<i>Test procedures</i> , 411	
a.	Union–intersection test, 411	
b.	Likelihood ratio test, 412	
c.	Other test statistics, 413	
d.	Comparison of four test statistics, 414	
e.	Mardia's permutation test, 416	
8.6.3	<i>Simultaneous confidence intervals</i> , 417	
8.6.4	<i>Comparing two populations</i> , 419	
8.6.5	<i>Canonical form</i> , 421	
8.6.6	<i>Missing observations</i> , 422	
8.6.7	<i>Other topics</i> , 423	
8.7	A Generalized Linear Hypothesis	423
8.7.1	<i>Theory</i> , 423	
8.7.2	<i>Profile analysis for K populations</i> , 424	
8.7.3	<i>Tests for mean of multivariate normal</i> , 425	
8.8	Step-Down Procedures	426
8.9	Multiple Design Models	428
	Exercises 8, 431	
9	MULTIVARIATE ANALYSIS OF VARIANCE AND COVARIANCE	433
9.1	Introduction	433
9.2	One-Way Classification	433
9.2.1	<i>Hypothesis testing</i> , 433	
9.2.2	<i>Multiple comparisons</i> , 437	
9.2.3	<i>Comparison of test statistics</i> , 440	
9.2.4	<i>Robust tests for equal means</i> , 443	
a.	Permutation test, 443	
b.	James' test, 445	
9.2.5	<i>Reparameterization</i> , 447	
9.2.6	<i>Comparing dispersion matrices</i> , 448	
a.	Test for equal dispersion matrices, 448	
b.	Graphical comparisons, 451	
9.2.7	<i>Exact procedures for means assuming unequal dispersion matrices</i> , 452	
a.	Hypothesis testing, 452	
b.	Multiple confidence intervals, 454	
9.3	Randomized Block Design	454
9.3.1	<i>Hypothesis testing and confidence intervals</i> , 454	
9.3.2	<i>Underlying assumptions</i> , 458	

9.4	Two-Way Classification With Equal Observations per Mean	458
9.5	Analysis of Covariance	463
9.5.1	<i>Univariate theory</i> , 463	
9.5.2	<i>Multivariate theory</i> , 465	
9.5.3	<i>Test for additional information</i> , 471	
9.6	Multivariate Cochran's Theorem on Quadratics	472
9.7	Growth Curve Analysis	474
9.7.1	<i>Examples</i> , 474	
a.	Single growth curve, 474	
b.	Two growth curves, 475	
c.	Single growth curve for a randomized block design, 476	
d.	Two-dimensional growth curve, 477	
9.7.2	<i>General theory</i> , 478	
a.	Potthoff and Roy's method, 479	
b.	Rao–Khatri analysis of covariance method, 480	
c.	Choice of method, 483	
9.7.3	<i>Single growth curve</i> , 484	
9.7.4	<i>Test for internal adequacy</i> , 486	
9.7.5	<i>A case study</i> , 487	
9.7.6	<i>Further topics</i> , 492	
	Exercises 9, 494	
10	SPECIAL TOPICS	496
10.1	Computational Techniques	496
10.1.1	<i>Solving the normal equations</i> , 496	
a.	Cholesky decomposition, 496	
b.	QR-algorithm, 497	
10.1.2	<i>Hypothesis matrix</i> , 498	
10.1.3	<i>Calculating Hotelling's T^2</i> , 499	
10.1.4	<i>Generalized symmetric eigenproblem</i> , 500	
a.	Multivariate linear hypothesis, 501	
b.	One-way classification, 503	
c.	Discriminant coordinates, 504	
10.1.5	<i>Singular value decomposition</i> , 504	
a.	Definition, 504	
b.	Solution of normal equations, 505	
c.	Principal components, 506	
d.	Canonical correlations, 506	
10.1.6	<i>Selecting the best subset</i> , 507	
a.	Response variables, 507	
b.	Regressor variables, 510	

10.2 Log-Linear Models for Binary Data	512
10.3 Incomplete Data	514

APPENDIX 517

A Some Matrix Algebra	517
A1 <i>Trace and eigenvalues</i> , 517	
A2 <i>Rank</i> , 518	
A3 <i>Patterned matrices</i> , 519	
A4 <i>Positive semidefinite matrices</i> , 521	
A5 <i>Positive definite matrices</i> , 521	
A6 <i>Idempotent matrices</i> , 522	
A7 <i>Optimization and inequalities</i> , 523	
A8 <i>Vector and matrix differentiation</i> , 530	
A9 <i>Jacobians and transformations</i> , 531	
A10 <i>Asymptotic normality</i> , 532	
B Orthogonal Projections	533
B1 <i>Orthogonal decomposition of vectors</i> , 533	
B2 <i>Orthogonal complements</i> , 534	
B3 <i>Projections on subspaces</i> , 535	
C Order Statistics and Probability Plotting	539
C1 <i>Sample distribution functions</i> , 539	
C2 <i>Gamma distribution</i> , 540	
C3 <i>Beta distribution</i> , 541	
C4 <i>Probability plotting</i> , 542	
D Statistical Tables	545
D1 <i>Bonferroni t-percentage points</i> , 546	
D2 <i>Maximum likelihood estimates for the gamma distribution</i> , 550	
D3 <i>Upper tail percentage points for $\sqrt{b_1}$</i> , 551	
D4 <i>Coefficients in a normalizing transformation of $\sqrt{b_1}$</i> , 551	
D5 <i>Simulation percentiles for b_2</i> , 553	
D6 <i>Charts for the percentiles of b_2</i> , 554	
D7 <i>Coefficients for the Wilk–Shapiro (W) test</i> , 556	
D8 <i>Percentiles for the Wilk–Shapiro (W) test</i> , 558	
D9 <i>D'Agostino's test for normality</i> , 558	
D10 <i>Anderson–Darling (A_n^2) test for normality</i> , 560	
D11 <i>Discordancy test for single gamma outlier</i> , 561	
D12 <i>Discordancy test for single multivariate normal outlier</i> , 562	
D13 <i>Wilks' likelihood ratio test</i> , 562	
D14 <i>Roy's maximum root statistic</i> , 563	
D15 <i>Lawley–Hotelling trace statistic</i> , 563	
D16 <i>Pillai's trace statistic</i> , 564	

D17	<i>Test for mutual independence</i> , 564	803
D18	<i>Test for equal dispersion matrices with equal sample sizes</i> , 564	803
 OUTLINE SOLUTIONS TO EXERCISES		 615
 REFERENCES		 626
 INDEX		 671