TABLE OF CONTENTS

Chapter 1

1.1 INTRODUCTION 1-2
1.2 EQUATIONS AND NUMBERS 1-2
1.2.1 Some equations with positive integers require negative solutions. . . 1-2
1.2.2 Some equations with integers require rational solutions. 1-8
1.2.3 Other equations with integers require real solutions. 1-14
1.2.4 Rational powers represent solutions of algebraic equations. 1-18
1.2.5 The algebra of real numbers contains that of rational numbers. . . . 1-22
1.3 AREAS AND SLOPES 1-28
1.3.1 Coordinate axes link numbers to graphs. 1-28
1.3.2 The axioms governing areas describe the physical world. 1-33
1.3.3 The slope of a straight line measures a rate of change. 1-40
1.4 MATHEMATICAL FUNCTIONS 1-46
1.4.1 Functions are algorithms, formulae, graphs, and tables. 1-46
1.5 SYNOPSIS 1-54
1.5.1 Algebra and analysis 1-54
1.5.2 Geometry 1-54
1.6 SELF-TEST 1-56
1.7 SOLUTIONS 1-57
1.7.1 Solutions to all the exercises 1-57
1.7.2 Solutions to the self-test 1-74
1.8 REFERENCES 1-75

Chapter 2

2.1 INTRODUCTION: WHY STUDY GRAPHS? 2-2
2.1.1 Graphs are information. 2-2
2.2 CASE STUDY: THE FEDERAL INCOME TAX RATES 2-2
2.2.1 The graph of a "function" pictures the tax schedule. 2-2
2.2.2 The "average" tax rate represents the fraction of income owed in taxes. 2-4
2.2.3 The "average rate of change" forms the basis for the "marginal" tax rate. 2-7
2.2.4 The "marginal" tax rate represents the tax on one additional dollar. . 2-10
2.2.5 The area "under" the marginal tax rate equals the tax amount. . . . 2-14
2.2.6 The 1986 tax law matched the upper average and marginal tax rates. 2-16
2.2.7 Previous federal income tax schedules had fifteen tax brackets. . . . 2-20
2.3 SYNOPSIS: A PREVIEW OF CALCULUS 2-23

2-25
2.4 SELF-TEST . . . 2-26
2.5 SOLUTIONS . . . 2-26
2.5.1 Solutions to all the exercises. . . . 2-35
2.5.2 Solution to the self-test. . . . 2-37
2.6 REFERENCES . . . 2-37

Chapter 3 . . . 3-2
3.1 INTRODUCTION . . . 3-2
3.1.1 The concept of "derivative" has many applications. . . . 3-2
3.1.2 The slope of the total-cost graph represents the marginal cost. . . . 3-4
3.2 MATHEMATICS . . . 3-4
3.2.1 Tangent lines stay "close" to curves. . . . 3-6
3.2.2 The slope of a curve is the slope of its tangent line. . . . 3-7
3.2.3 The slope approximates the vertical change per unit horizontal change. . . . 3-8
3.2.4 The derivative records slopes. . . . 3-10
3.2.5 Sketches of derivatives often suffice. . . . 3-12
3.3 APPLICATIONS . . . 3-13
3.3.1 The marginal cost represents the rate of change of the total cost. . . . 3-15
3.3.2 At its minimum the average cost equals the marginal cost. . . . 3-18
3.3.3 At the highest profit the marginal cost equals the marginal revenue. . . . 3-20
3.3.4 "Should" price equal average cost or marginal cost? . . . 3-22
3.4 SYNOPSIS . . . 3-23
3.5 SELF-TEST . . . 3-25
3.6 SOLUTIONS . . . 3-25
3.6.1 Solutions to all the exercises . . . 3-29
3.6.2 Solution to the self-test . . . 3-32
3.7 REFERENCES . . . 3-32

Chapter 4 . . . 4-2
4.1 INTRODUCTION . . . 4-2
4.2 MATHEMATICS: THE FUNDAMENTAL THEOREM OF CALCULUS . . . 4-3
4.2.1 Squares approximate curvilinear areas. . . . 4-3
4.2.2 Rectangles approximate the area bounded by a function. . . . 4-7
4.2.3 The derivative of the integral is the initial function. . . . 4-16
4.3 APPLICATION: OPTIMAL PRICING POLICIES . . . 4-26

4.3.1 The area under the supply curve represents the total variable cost . . 4-26
4.3.2 The area under the demand curve represents the maximum total revenue. 4-30
4.3.4 The area between demand and supply represents the combined surplus. 4-34
4.5 CASE-STUDY: ASSESSING THE PERFORMANCE OF MANAGEMENT 4-36
4.5.1 Does the management give the public the most for its money? . . . 4-37
4.5.2 The management appears to maximize the number of spectators. . . 4-38
4.6 SELF-TEST 4-39
4.7 SOLUTIONS 4-41
4.7.1 Solutions to all the exercises 4-41
4.7.2 Solution to the self-test. 4-48
4.8 REFERENCES 4-50

Chapter 5

5.1 INTRODUCTION 5-2
5.2 MATHEMATICS: LINEAR EQUATIONS AND STRAIGHT LINES 5-3
5.2.1 Linear equations and straight lines correspond to one another. . . . 5-3
5.2.2 Different forms of the same equation fit different needs. 5-6
5.2.3 To graph linear equations, solve and plot two points. 5-11
5.2.4 Linear inequalities correspond to half planes. 5-15
5.2.5 Linear systems correspond to intersections of straight lines. 5-17
5.3 APPLICATIONS: MARKET EQUILIBRIUM AND YIELD OF BILLS 5-22
5.3.1 Supply and demand meet at an equilibrium point. 5-22
5.3.2 The yield of short term Treasury bills obeys linear equations. 5-24
5.4 SYNOPSIS 5-26
5.5 SELF-TEST 5-28
5.6 SOLUTIONS 5-30
5.6.1 Solutions to all the exercises. 5-30
5.6.2 Solution to the self-test 5-38
5.7 REFERENCES 5-39

Chapter 6

6.1 INTRODUCTION: HOW CAN LINEAR PROGRAMMING HELP IN BUSINESS? 6-2

6.2 GRAPHIC SOLUTION: "OPTIMAL" CASE-MIX FOR A HOSPITAL 6-2
6.2.1 Each constraint translates into an inequality. 6-3
6.2.2 Each inequality corresponds to a half-plane bounded by a straight line. 6-4
6.2.3 The quantity to be optimized stays constant along some parallel lines. 6-7
6.2.4 The optimal quantity occurs at a corner of the feasible region. . . . 6-8
6.3 COMPUTATIONAL SOLUTIONS THROUGH LINEAR ALGEBRA 6-10
6.3.1 Testing all corners solves linear programming problems, but slowly. . 6-10
6.3.2 Computers would have to check each intersection. 6-12
6.3.3 Fourier's method solves inequalities algebraically. 6-14
6.4 SYNOPSIS 6-17
6.5 SELF-TEST 6-18
6.6 SOLUTIONS 6-19
6.6.1 Solutions to all the exercises 6-19
6.6.2 Solution to the self-test. 6-24
6.7 REFERENCES 6-26

Chapter 7

7.1 INTRODUCTION 7-2
7.2 MATHEMATICS: QUADRATIC EQUATIONS AND PARABOLAE 7-3
7.2.1 The graph of the simplest quadratic equation "falls" and then "rises." 7-3
7.2.2 Completing the square reveals the vertex for a quadratic equation. . 7-5
7.2.3 The quadratic formula solves quadratic equations. 7-9
7.2.4 The derivative of a quadratic function obeys a linear equation. . . . 7-14
7.2.5 Synopsis: plot the vertex, intercepts, points, and tangents of parabolae. 7-20
7.3 APPLICATION TO FINANCE: YIELD OF ONE-YEAR TREASURY BILLS 7-23
7.3.1 The quadratic formula gives the yield rate of Treasury bills. 7-23
7.4 APPLICATION TO MICROECONOMICS: MAXIMUM PROFIT . 7-28
7.4.1 With a linear demand, revenues follow a quadratic equation. 7-28
7.4.2 Economic "marginal" quantities are derivatives. 7-32
7.5 CASE-STUDY: THE SHAKESPEARE MEMORIAL THEATRE . . 7-36
7.5.1 Question: does the Theatre maximize profit, revenue, or attendance? 7-36
7.5.2 Answer: the management maximizes the number of attendances. . . 7-36