This book is an introduction to the techniques of many-body quantum theory with a large number of applications to condensed matter physics. The basic idea of the book is to provide a self-contained formulation of the theoretical framework without losing mathematical rigor, while at the same time providing physical motivation and examples. The examples are taken from applications in electron systems and transport theory.

On the formal side, the book covers an introduction to second quantization, many-body Green's function, finite temperature Feynman diagrams and bosonization. The applications include traditional transport theory in bulk as well as mesoscopic systems, where both the Landauer-Büttiker formalism and recent developments in correlated transport phenomena in mesoscopic systems and nano-structures are covered. Other topics include interacting electron gases, plasmons, electron-phonon interactions, superconductivity and one-dimensional systems, and Luttinger liquid theory.

Having grown out of a set of lecture notes, incorporating many pedagogical exercises, this book is designed as a textbook for an advanced undergraduate or graduate course, but is also well suited for self-study.

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