

## **Ergodic theory**

KARL PETERSEN

The study of dynamical systems forms a vast and rapidly developing field even when one considers only activity whose methods derive mainly from measure theory and functional analysis. Karl Petersen has written a book which presents the fundamentals of the ergodic theory of point transformations and then several advanced topics which are currently undergoing intense research. By selecting one or more of these topics to focus on, the reader can quickly approach the specialised literature and indeed the frontier of the area of interest. Each of the four basic aspects of ergodic theory – examples, convergence theorems, recurrence properties, and entropy – receives first a basic and then a more advanced, particularised treatment. At the introductory level, the book provides clear and complete discussions of the standard examples, the mean and pointwise ergodic theorems, recurrence, ergodicity, weak mixing, strong mixing, and the fundamentals of entropy. Among the advanced topics are a thorough treatment of maximal functions and their usefulness in ergodic theory, analysis, and probability, an introduction to almost periodic functions and topological dynamics, a proof of the Jewett–Krieger theorem, an introduction to multiple recurrence and the Szemerédi–Furstenberg theorem, and the Keane–Smorodinsky proof of Ornstein’s isomorphism theorem for Bernoulli shifts.

The author’s easily-readable style combined with the profusion of exercises and references, summaries, historical remarks, and heuristic discussions make this book useful either as a text for graduate students or self-study, or as a reference work for the initiated.

**Cambridge University Press**

0 521 24407 2

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