

Differential Forms

There already exist a number of excellent graduate textbooks on the theory of differential forms as well as a handful of very good undergraduate textbooks on multivariable calculus in which this subject is briefly touched upon but not elaborated on enough.

The goal of this textbook is to be readable and usable for undergraduates. It is entirely devoted to the subject of differential forms and explores a lot of its important ramifications.

In particular, our book provides a detailed and lucid account of a fundamental result in the theory of differential forms which is, as a rule, not touched upon in undergraduate texts: the isomorphism between the Čech cohomology groups of a differential manifold and its de Rham cohomology groups.

- **Authoritative** textbook on differential forms for undergraduates
- Includes numerous **Examples** and **Exercises** for further in-depth understanding on the presented concepts
- The first author, **Victor Guillemin**, is a world-renowned mathematician in the field of symplectic geometry
- His co-author, **Peter Haine**, is a talented doctoral student at MIT under Clark Barwick. His research interests center around homotopy theory, algebraic K -theory and algebraic geometry

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