

# METHODS OF QUANTUM FIELD THEORY IN STATISTICAL PHYSICS

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Revised English Edition Translated and Edited by Richard A. Silverman

**L**ucidly written and systematic in treatment, this book is a prominent work on many-body theory and its ramifications. Written by three members of the Institute for Physical Problems of the Academy of Sciences of the U.S.S.R., this work has been translated and revised by editor Richard A. Silverman, noted for his many translations of Russian works in the mathematical sciences.

Using Green's functions for their basic methodological approach, the authors develop their material in seven connected chapters. The first chapter contains a preliminary discussion of several basic topics, including elementary excitations, the Fermi liquid, and second quantization. Chapters 2 and 3 present parallel methods of quantum field theory for  $T=0$  and  $T\neq 0$ , both involving the diagram technique. The final four chapters apply the technique and other information learned here to a discussion of the theory of the Fermi liquid, systems of interacting bosons, electromagnetic radiation in an absorbing medium, and the theory of superconductivity. Among the topics considered in these final chapters are electron-photon interactions, some properties of a degenerate plasma, the dilute nonideal Bose gas, properties of the spectrum of one-particle excitations near the cutoff point, molecular interaction forces, the basic system of equations for a superconductor, and the theory of superconducting alloys.

This basic text is indispensable to scientists, physicists, and mathematicians involved with statistical physics, in particular cryogenics and solid state physics. The only prerequisite is a familiarity with statistical physics and quantum mechanics.

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