Crystallography is an interdisciplinary science covering a wide area, from biology to earth sciences, mathematics, and materials science. Its role is growing, owing to the contribution crystallography can offer to the understanding of such diverse fields as biological structures, high-temperature superconductors, mineral properties, and phase transitions. This book describes both the theoretical bases and applications of different areas interacting with crystallography. As with the first and second editions, it is organized as a collection of chapters written by recognized specialists, with all contributions being harmonized into a unified whole. The main text is devoted to the presentation of basics; appendices to the chapters deal with specialist aspects. In this third edition topics have been updated so as to document the present state-of-the-art: emphasis is placed upon areas of current research.

- Updated throughout to reflect the current state-of-the-art, and for clearer pedagogy.
- 95 new figures introduced, to aid understanding.
- Reference lists updated to reflect recent advances in the field.
- Many sections completely rewritten, including those on powder crystallography, electron diffraction, phase determination procedures using dual space approaches, and H-bonding.
- New material introduced, including sections on charge-transfer or donor–acceptor interactions, electron density modification methods, and humidity control of macromolecular crystals.

The complimentary CD of the second edition is now available via the web: http://wwwba.ic.cnr.it/abc. Modern graphics will help users to understand the basics of this science via three-dimensional images, simulation of experiments, and exercises.

From reviews of previous editions:

'A new research worker could turn to the appropriate chapter and find a description of how to tackle real problems ... this book is an excellent crystallography text, both for those involved with the subject in a practical sense or those that require fundamental information on the subject ... it has that rare quality that the more it is read, the better it seems. It deserves a place on the shelf of all those involved in, or interested in, crystallography.' *Materials World*

'This is a remarkable text. It is a tribute at once to the wide reach of crystallography over many branches of science and to the contemporary vigor and vitality of Italian crystallography and crystallographers.' *Acta Crystallographica*

'... there is a good balance achieved between well-illustrated description and mathematical rigour. Established crystallographers will find it a handy reference manual, while at the same time the text remains accessible and informative reading for undergraduates with an interest in crystallography.' *The Times Higher Education Supplement*

Cover image: Cartoon representation of the folding of NAPA from *Helicobacter pylori*, a bacterium that lives in the human stomach. 12 identical monomers, arranged in a nearly spherical shell with 23 symmetry, form an internal cavity where iron can be stored. The space filling model of the protein is shown in the background. Small red spheres represent coordinated Fe ions (Protein Data Bank code 1JI4).

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Carmelo Giacovazzo

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