	List of Figures List of Tables	ix xiii
	Foreword	XV
	Preface	xvii
	Preface to the Fourth Edition	xix
1	Introduction to Fuzzy Sets	1
1.1	Crispness, Vagueness, Fuzziness, Uncertainty	1
1.2	Fuzzy Set Theory	2
Part I:	Fuzzy Mathematics	9
2	Fuzzy Sets—Basic Definitions	11
2.1	Basic Definitions	11
2.2	Basic Set-Theoretic Operations for Fuzzy Sets	16
3	Extensions	23
3.1	Types of Fuzzy Sets	23
3.2	Further Operations on Fuzzy Sets	27
3.2.1	Algebraic Operations	28
3.2.2	Set-Theoretic Operations	29
3.2.3	Criteria for Selecting Appropriate Aggregation Operators	43
4	Fuzzy Measures and Measures of Fuzziness	47
4.1	Fuzzy Measures	47
4.2	Measures of Fuzziness	49
5	The Extension Principle and Applications	55
5.1	The Extension Principle	55
5.2	Operations for Type 2 Fuzzy Sets	56
5.3	Algebraic Operations with Fuzzy Numbers	59
5.3.1	Special Extended Operations	61
5.3.2	Extended Operations for LR-Representation of Fuzzy Sets	64

6 6.1 6.1.1 6.1.2 6.2 6.3	Fuzzy Relations and Fuzzy Graphs Fuzzy Relations on Sets and Fuzzy Sets Compositions of Fuzzy Relations Properties of the Min-Max Composition Fuzzy Graphs Special Fuzzy Relations	71 71 76 79 83 86
7 7.1 7.2 7.3 7.3.1 7.3.2 7.4	Fuzzy Analysis Fuzzy Functions on Fuzzy Sets Extrema of Fuzzy Functions Integration of Fuzzy Functions Integration of a Fuzzy Function over a Crisp Interval Integration of a (Crisp) Real-Valued Function over a Fuzzy Interval Fuzzy Differentiation	93 93 95 99 100 103 107
8 8.1 8.1.1 8.1.2 8.1.3 8.1.4 8.1.5 8.2 8.2.1 8.2.2 8.3 8.3.1 8.3.2 8.4	Uncertainty Modeling Application-oriented Modeling of Uncertainty Causes of Uncertainty Type of Available Information Uncertainty Methods Uncertainty Theories as Transformers of Information Matching Uncertainty Theory and Uncertain Phenomena Possibility Theory Fuzzy Sets and Possibility Distributions Possibility and Necessity Measures Probability of Fuzzy Events Probability of a Fuzzy Event as a Scalar Probability of a Fuzzy Event as a Fuzzy Set Possibility vs. Probability	111 111 114 117 118 119 120 122 122 126 129 129 131 133
Part II:	Applications of Fuzzy Set Theory	139
9.1 9.2 9.2.1 9.2.2 9.3 9.4 9.5 9.5.1 9.5.2 9.5.3 9.5.4 9.5.5 9.5.6 9.5.7	Fuzzy Logic and Approximate Reasoning Linguistic Variables Fuzzy Logic Classical Logics Revisited Linguistic Truth Tables Approximate and Plausible Reasoning Fuzzy Languages Support Logic Programming and Fril Introduction Fril Rules Inference Methods in Fril Fril Inference for a Single Rule Multiple Rule Case Interval and Point Semantic Unification Least Prejudiced Distribution and Learning	141 149 149 153 156 160 169 170 172 175 176 177
9.5.8	Applications of Fril	181

CONTI	ENTS	vii
10	Fuzzy Sets and Expert Systems	105
10.1	Introduction to Export Customs	185
10.2	Incortainty Madeline in F	185
10.3	Applications	193
		203
11	Fuzzy Control	
11.1	Origin and Objective	223
11.2	Automatic Control	225
11.3	The Fuzzy Controller	226
11.4	Types of Fuzzy Controllers	228
11.4.1	The Mamdani Controller	228
11.4.2	Defuzzification	232
11.4.3	The Sugeno Controller	239
11.5	Design Parameters	
11.5.1	Scaling Factors	240
11.5.2	Fuzzy Sets	240
11.5.3	Rules	242
11.6	Adaptive Fuzzy Control	243
11.7	Applications	244
11.7.1	Crane Control	244
11.7.2	Control of a Model Car	246
11.7.3	Control of a Diesel Engine	
11.7.4	Fuzzy Control of a Cement Kiln	248
11.8	Tools	249
11.9	Stability	255
11.10	Extensions	257 262
12	Europe Data Danas de la companya del companya de la companya del companya de la c	
12.1	Fuzzy Data Bases and Queries	265
12.2	Introduction	265
12.3	Fuzzy Relational Databases	266
12.3	Fuzzy Queries in Crisp Databases	268
13	Fuzzy Data Analysis	
13.1	Inducational and	277
13.2	Methods for Fuzzy Data Analysis	
13.2.1	Algorithmic Approaches	279
13.2.2	V	281
13.2.3		002
13.3	Neural Net Approaches Dynamic Fuzzy Data Analysis	
13.3.1	Problem Description	306
13.3.2	Similarity of Functions	306
13.3.3	Approaches for Analysic Dynamic Systems	307
13.4	Tools for Fuzzy Data Analysis	313
13.4.1	Requirements for FDA Tools	317
13.4.2	Data Engine	317
13.5	Applications of FDA	318
13.5.1	Maintenance Management in Detroit of De	322
13.5.2	Maintenance Management in Petrochemical Plants Acoustic Quality Control	322
	Accusio Quality Collifor	323

14	Decision Making in Fuzzy Environments	329
14.1	Fuzzy Decisions	329
14.2	Fuzzy Linear Programming	336
14.2.1	Symmetric Fuzzy LP	337
14.2.2	Fuzzy LP with Crisp Objective Function	342
14.3	Fuzzy Dynamic Programming	
14.3.1	Fuzzy Dynamic Programming with Crisp State Transformation	348
14.0.1	Function	040
14.4		349
14.4.1	Fuzzy Multicriteria Analysis	352
	Multi Objective Decision Making (MODM)	353
14.4.2	Multi Attributive Decision Making (MADM)	359
15	Applications of Fuzzy Sets in Engineering and Management	371
15.1	Introduction	371
15.2	Engineering Applications	373
15.2.1	Linguistic Evaluation and Ranking of Machine Tools	375
15.2.2	Fault Detection in Gearboxes	381
15.3	Applications in Management	
15.3.1	A Discrete Location Model	389
15.3.2	Fuzzy Set Models in Logistics	390
15.3.2.1		393
15.3.2.2	Fuzzy Approach to the Transportation Problem	393
15.3.3	Fuzzy Linear Programming in Logistics	398
	Fuzzy Sets in Scheduling	401
15.3.3.1	Job-Shop Scheduling with Expert Systems	401
15.3.3.2	A Method to Control Flexible Manufacturing Systems	405
15.3.3.3	Aggregate Production and Inventory Planning	411
15.3.3.4	Fuzzy Mathematical Programming for Maintenance Scheduling	418
15.3.3.5	Scheduling Courses, Instructors, and Classrooms	419
15.3.4	Fuzzy Set Models in Inventory Control	426
15.3.5	Fuzzy Sets in Marketing	432
15.3.5.1	Customer Segmentation in Banking and Finance	432
15.3.5.2	Bank Customer Segmentation based on Customer Behavior	433
16	Empirical Research in Fuzzy Set Theory	443
16.1	Formal Theories vs. Factual Theories vs. Decision Technologies	
16.1.1	Models in Operations Passarch and Management Science	443
16.1.2	Models in Operations Research and Management Science Testing Factual Models	447
16.2		449
	Empirical Research on Membership Functions	453
16.2.1	Type A-Membership Model	454
16.2.2	Type B-Membership Model	456
16.3	Empirical Research on Aggregators	463
16.4	Conclusions	474
17	Future Perspectives	477
	Abbreviations of Frequently Cited Journals	481
	Bibliography	483
	Index	507
Carrier To		001