Statistical Theory and Methods

Optimal Experimental Design with R

Experimental design is often overlooked in the literature of applied and mathematical statistics: statistics is taught and understood as merely a collection of methods for analyzing data. Consequently, experimenters seldom think about optimal design, including prerequisites such as the necessary sample size needed for a precise answer for an experimental question.

Providing a concise introduction to experimental design theory, **Optimal Experimental Design with R**:

- Introduces the philosophy of experimental design
- Provides an easy process for constructing experimental designs and calculating necessary sample size using R programs
- Teaches by example using a custom made R program package: OPDOE

Consisting of detailed, data-rich examples, this book introduces experimenters to the philosophy of experimentation, experimental design, and data collection. It gives researchers and statisticians guidance in the construction of optimum experimental designs using R programs, including sample size calculations, hypothesis testing, and confidence estimation. A final chapter of in-depth theoretical details is included for interested mathematical statisticians.

CRC Press
Taylor & Francis Group
an **informa** business
www.crcpress.com

ISBN 978-0-367-38276-6

9 780367 382766

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