

Branching Processes

Variation, Growth, and
Extinction of Populations



Paperback Re-issue

Biology takes a special place among the other natural sciences because biological units, be they pieces of DNA, cells, or organisms, reproduce more or less faithfully. Like any other biological process, reproduction has a large random component. The theory of branching processes was developed especially as a mathematical counterpart to this most fundamental of biological processes. This active and rich research area allows us to determine extinction risks and predict the development of population composition, and also uncover aspects of a population's history from its current genetic composition. Branching processes play an increasingly important role in models of genetics, molecular biology, microbiology, ecology, and evolutionary theory. This book presents this body of mathematical ideas for a biological audience, but should also be enjoyable to mathematicians, if only for its rich stock of realistic biological examples. It can be read by anyone with a basic command of calculus, matrix algebra, and probability theory. More advanced results from basic probability theory are treated in a special appendix.



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