

TEXTBOOKS IN MATHEMATICS

An Introduction to Mathematical Proofs presents fundamental material on logic, proof methods, set theory, number theory, relations, functions, cardinality, and the real number system. The text uses a methodical, detailed, and highly structured approach to proof techniques and related topics. No prerequisites are needed beyond high-school algebra.

New material is presented in small chunks that are easy for beginners to digest. The author offers a friendly style without sacrificing mathematical rigor. Ideas are developed through motivating examples, precise definitions, carefully stated theorems, clear proofs, and a continual review of preceding topics.

Features

- Study aids including section summaries and over 1100 exercises
- Careful coverage of individual proof-writing skills
- Proof annotations and structural outlines clarify tricky steps in proofs
- Thorough treatment of multiple quantifiers and their role in proofs
- Unified explanation of recursive definitions and induction proofs, with applications to greatest common divisors and prime factorizations

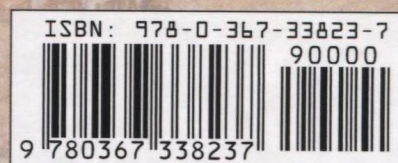
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