

Leibniz Algebras: Structure and Classification is designed to introduce the reader to the theory of Leibniz algebras.

Leibniz algebra is the generalization of Lie algebras. These algebras preserve a unique property of Lie algebras that the right multiplication operators are derivations. They first appeared in papers of A.M. Blokh in the 1960s, under the name D-algebras, emphasizing their close relationship with derivations. The theory of D-algebras did not get as thorough an examination as it deserved immediately after its introduction. Later, the same algebras were introduced in 1993 by Jean-Louis Loday, who called them Leibniz algebras due to the identity they satisfy. The main motivation for the introduction of Leibniz algebras was to study the periodicity phenomena in algebraic K-theory.

Nowadays, the theory of Leibniz algebras is one of the more actively developing areas of modern algebra. Along with (co)homological, structural and classification results on Leibniz algebras, some papers with various applications of the Leibniz algebras also appear now. However, the focus of this book is mainly on the classification problems of Leibniz algebras. Particularly, the authors propose a method of classification of a subclass of Leibniz algebras based on algebraic invariants. The method is applicable in the Lie algebras case as well.

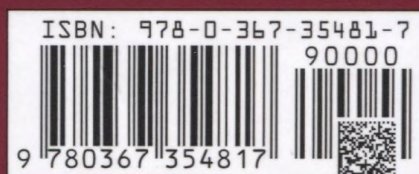
Features

- Provides a systematic exposition of the theory of Leibniz algebras and recent results on Leibniz algebras
- Suitable for final year bachelor's, master's and PhD students going into research in the structural theory of finite-dimensional algebras, particularly, Lie and Leibniz algebras
- Covers important and more general parts of the structural theory of Leibniz algebras that are not addressed in other texts

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