A new look at weak-convergence methods in metric spaces—from a master of probability theory

In this new edition, Patrick Billingsley updates his classic work *Convergence of Probability Measures* to reflect developments of the past thirty years. Widely known for his straightforward approach and reader-friendly style, Dr. Billingsley presents a clear, precise, up-to-date account of probability limit theory in metric spaces. He incorporates many examples and applications that illustrate the power and utility of this theory in a range of disciplines—from analysis and number theory to statistics, engineering, economics, and population biology.

With an emphasis on the simplicity of the mathematics and smooth transitions between topics, the *Second Edition* boasts major revisions of the sections on dependent random variables as well as new sections on relative measure, on lacunary trigonometric series, and on the Poisson-Dirichlet distribution as a description of the long cycles in permutations and the large divisors of integers. Assuming only standard measure-theoretic probability and metric-space topology, *Convergence of Probability Measures* provides statisticians and mathematicians with basic tools of probability theory as well as a springboard to the "industrial-strength" literature available today.

PATRICK BILLINGSLEY, PhD, is Professor of Mathematics and Statistics at the University of Chicago. His book, *Probability and Measure, Third Edition*, is also available from Wiley.

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