

Point Processes

D.R. Cox and Valerie Isham

There has been much recent research on the theory of point processes, i.e., on random systems consisting of point events occurring in space or time. Applications range from emissions from a radioactive source, occurrences of accidents or machine breakdowns, or of electrical impulses along nerve fibres, to repetitive point events in an individual's medical or social history. Sometimes the point events occur in space rather than time and the applications here range from statistical physics to geography. The object of this book is to develop the applied mathematics of point processes at a level which will make the ideas accessible both to the research worker and postgraduate student in probability and statistics and also to the mathematically inclined individual in another field interested in using the ideas and results. A thorough knowledge of the key notions of elementary probability theory is required to understand the book, but specialized "pure mathematical" considerations have been avoided.

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