

The Theory of Stochastic Processes

D. R. Cox and H. D. Miller

The theory of stochastic processes is concerned with systems which change in accordance with probabilistic laws. There are applications in many branches of the physical and biological sciences and in technology. The present book develops the main mathematical techniques useful in analysing the special processes arising in applications. The reader is assumed to know some elementary probability theory and matrix algebra and advanced calculus.

The book is intended partly for students of statistics and mathematics and partly for research workers encountering problems in applied probability.

The chapters are as far as possible self-contained and each contains material of varied difficulty, starting always with simple examples of special processes. Therefore, if used as a textbook, there will be freedom to choose material depending on the students' particular interests and mathematical knowledge. There are exercises and suggestions for further reading at the end of each chapter.

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From the reviews

"This is an important book which will also, I believe, be very successful . . . it is a carefully written and illuminating account of stochastic processes, written at a level which will make it useful to a large class of readers, certain as a consequence to be widely read, and thus a work of considerable importance."

E. J. Hannan, *The Australian Journal of Statistics*

"Here is a clear and readable exposition of everything in stochastic process theory that the non-specialist is likely to want to know; and for those who wish to probe more deeply there is an excellent bibliography, with bibliographic notes at the end of each chapter. The authors' stated intention was to write a book which could serve the needs of both students and research workers. In this way they have succeeded admirably; this is a book which post-graduate students will enjoy reading and, because the emphasis is on methods rather than general theory, the research worker will find it a useful reference book. All the important topics are discussed, both in general terms and in terms of examples drawn from many branches of science and technology, and one of the attractions of the book is the way in which the various topics are shown to hang together."

J. M. Howl, *Control, ASLIB Book List*

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