

Principles of Copula Theory explores the state of the art on copulas and provides you with the foundation to use copulas in a variety of applications. Throughout the book, historical remarks and further readings highlight active research in the field, including new results, streamlined presentations, and new proofs of old results.

After covering the essentials of copula theory, the book addresses the issue of modelling dependence among components of a random vector using copulas. It then presents copulas from the point of view of measure theory, compares methods for the approximation of copulas, and discusses the Markov product for 2-copulas. The authors also examine selected families of copulas that possess appealing features from both theoretical and applied viewpoints. The book concludes with in-depth discussions on two generalisations of copulas: quasi- and semi-copulas.

Features

- Presents the theory of general d -dimensional copulas
- Unifies various methods scattered throughout the literature in common frameworks, including shuffles of copulas
- Proves the results as simply as possible
- Explores connexions with related functions, such as quasi-copulas, semi-copulas, and triangular norms, that have been used in different domains
- Demonstrates the importance of copulas in applied fields

Although copulas are not the solution to all stochastic problems, they are an indispensable tool for understanding several problems about stochastic dependence. This book gives you the solid and formal mathematical background to apply copulas to a range of mathematical areas, such as probability, real analysis, measure theory, and algebraic structures.



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