

Fractional Calculus and Fractional Processes with Applications to Financial Economics

Theory and Application

A comprehensive, go-to reference that explains fractional calculus, fractional processes, and their applications to financial economics

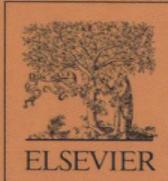
Fractional Calculus and Fractional Processes with Applications to Financial Economics: Theory and Application presents the theory and application of fractional calculus and fractional processes to financial data. Fractional calculus dates back to 1695 when Gottfried Wilhelm Leibniz first suggested the possibility of fractional derivatives. Research on fractional calculus started in full earnest in the second half of the twentieth century. The fractional paradigm applies not only to calculus, but also to stochastic processes, used in many applications in financial economics such as modeling volatility, interest rates, and high-frequency data. The key features of fractional processes that make them interesting are long-range memory, path-dependence, non-Markovian properties, self-similarity, fractal paths, and anomalous diffusion behavior. In this book, the authors discuss how fractional calculus and fractional processes are used in financial modeling and financial economic theory. It provides a practical guide that can be useful for students, researchers, and quantitative asset managers and risk managers interested in applying fractional calculus and fractional processes to asset pricing, financial time-series analysis, stochastic volatility modeling, and portfolio optimization.

- Provides the necessary background for applying fractional calculus to financial economics
- Analyzes the application of fractional calculus and fractional processes from deterministic and stochastic perspectives

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