

Michael S. Hamada • Alyson G. Wilson C. Shane Reese • Harry F. Martz **Bayesian Reliability**

Bayesian Reliability presents modern methods and techniques for analyzing reliability data from a Bayesian perspective. The adoption and application of Bayesian methods in virtually all branches of science and engineering have significantly increased over the past few decades. This increase is largely due to advances in simulation-based computational tools for implementing Bayesian methods.

The authors extensively use such tools throughout this book, focusing on assessing the reliability of components and systems with particular attention to hierarchical models and models incorporating explanatory variables. Such models include failure time regression models, accelerated testing models, and degradation models. The authors pay special attention to Bayesian goodness-of-fit testing, model validation, reliability test design, and assurance test planning. Throughout the book, the authors use Markov chain Monte Carlo (MCMC) algorithms for implementing Bayesian analyses – algorithms that make the Bayesian approach to reliability computationally feasible and conceptually straightforward.

This book is primarily a reference collection of modern Bayesian methods in reliability for use by reliability practitioners. There are more than 70 illustrative examples, most of which utilize real-world data. This book can also be used as a textbook for a course in reliability and contains more than 160 exercises.

Noteworthy highlights of the book include Bayesian approaches for the following:

- Goodness-of-fit and model selection methods
- Hierarchical models for reliability estimation
- Fault tree analysis methodology that supports data acquisition at all levels in the tree
- Bayesian networks in reliability analysis
- Analysis of failure count and failure time data collected from repairable systems, and the assessment of various related performance criteria
- Analysis of nondestructive and destructive degradation data
- Optimal design of reliability experiments
- Hierarchical reliability assurance testing

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Preface	VII
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1 Reliability Concepts	1
1.1 Defining Reliability	1
1.2 Measures of Random Variation	2
1.3 Examples of Reliability Data	10
1.3.1 Bernoulli Success/Failure Data	10
1.3.2 Failure Count Data	10
1.3.3 Lifetime/Failure Time Data	11
1.3.4 Degradation Data	12
1.4 Censoring	13
1.5 Bayesian Reliability Analysis	15
1.6 Related Reading	18
1.7 Exercises for Chapter 1	19
2 Bayesian Inference	21
2.1 Introductory Concepts	21
2.1.1 Maximum Likelihood Estimation	24
2.1.2 Classical Point and Interval Estimation for a Proportion	26
2.2 Fundamentals of Bayesian Inference	27
2.2.1 The Prior Distribution	28
2.2.2 Combining Data with Prior Information	30
2.3 Prediction	35
2.4 The Marginal Distribution of the Data and Bayes' Factors	36
2.5 A Lognormal Example	39
2.6 More on Prior Distributions	46
2.6.1 Noninformative and Diffuse Prior Distributions	46
2.6.2 Conjugate Prior Distributions	47
2.6.3 Informative Prior Distributions	47
2.7 Related Reading	49
2.8 Exercises for Chapter 2	49

3	Advanced Bayesian Modeling and Computational Methods	51
3.1	Introduction to Markov Chain Monte Carlo (MCMC)	51
3.1.1	Metropolis-Hastings Algorithms	52
3.1.2	Gibbs Sampler	60
3.1.3	Output Analysis	64
3.2	Hierarchical Models	68
3.2.1	MCMC Estimation of Hierarchical Model Parameters	71
3.2.2	Inference for Launch Vehicle Probabilities	71
3.3	Empirical Bayes	73
3.4	Goodness of Fit	76
3.5	Related Reading	82
3.6	Exercises for Chapter 3	82
4	Component Reliability	85
4.1	Introduction	85
4.2	Discrete Data Models for Reliability	86
4.2.1	Success/Failure Data	86
4.2.2	Failure Count Data	87
4.3	Failure Time Data Models for Reliability	90
4.3.1	Exponential Failure Times	91
4.3.2	Weibull Failure Times	97
4.3.3	Lognormal Failure Times	102
4.3.4	Gamma Failure Times	104
4.3.5	Inverse Gaussian Failure Times	105
4.3.6	Normal Failure Times	106
4.4	Censored Data	107
4.5	Multiple Units and Hierarchical Modeling	111
4.6	Model Selection	116
4.6.1	Bayesian Information Criterion	116
4.6.2	Deviance Information Criterion	117
4.6.3	Akaike Information Criterion	120
4.7	Related Reading	120
4.8	Exercises for Chapter 4	120
5	System Reliability	125
5.1	System Structure	125
5.1.1	Reliability Block Diagrams	126
5.1.2	Structure Functions	126
5.1.3	Minimal Path and Cut Sets	129
5.1.4	Fault Trees	131
5.2	System Analysis	135
5.2.1	Calculating System Reliability	135
5.2.2	Prior Distributions for Systems	138
5.2.3	Fault Trees with Bernoulli Data	141

5.2.4	Fault Trees with Lifetime Data	145
5.2.5	Bayesian Network Models	147
5.2.6	Models for Dependence	155
5.3	Related Reading	158
5.4	Exercises for Chapter 5	159
6	Repairable System Reliability	161
6.1	Introduction	161
6.1.1	Types of Data	162
6.1.2	Characteristics of System Repairs	162
6.2	Renewal Processes	163
6.3	Poisson Processes	165
6.3.1	Homogeneous Poisson Processes (HPPs)	167
6.4	Nonhomogeneous Poisson Processes (NHPPs)	170
6.4.1	Power Law Processes (PLPs)	170
6.4.2	Log-Linear Processes	176
6.5	Alternatives to NHPPs	176
6.5.1	Modulated Power Law Processes (MPLPs)	176
6.5.2	Piecewise Exponential Model (PEXP)	179
6.6	Goodness of Fit and Model Selection	180
6.7	Current Reliability and Other Performance Criteria	181
6.7.1	Current Reliability	181
6.7.2	Other Performance Criteria	182
6.8	Multiple-Unit Systems and Hierarchical Modeling	183
6.9	Availability	192
6.9.1	Other Data Types for Availability	194
6.9.2	Complex System Availability	196
6.10	Related Reading	198
6.11	Exercises for Chapter 6	199
7	Regression Models in Reliability	203
7.1	Introduction	203
7.1.1	Covariate Types	204
7.1.2	Covariate Relationships	205
7.2	Logistic Regression Models for Binomial Data	205
7.3	Poisson Regression Models for Count Data	215
7.4	Regression Models for Lifetime Data	221
7.5	Model Selection	228
7.6	Residual Analysis	229
7.7	Accelerated Life Testing	235
7.7.1	Common Accelerating Variables and Relationships	237
7.8	Reliability Improvement Experiments	243
7.9	Other Regression Situations	258
7.10	Related Reading	259
7.11	Exercises for Chapter 7	259

8	Using Degradation Data to Assess Reliability	27
8.1	Introduction	27
8.1.1	Comparison with Lifetime Data	27
8.2	More Complex Degradation Data Models	27
8.2.1	Reliability Function	28
8.3	Diagnostics for Degradation Data Models	28
8.4	Incorporating Covariates	28
8.4.1	Accelerated Degradation Testing	28
8.4.2	Improving Reliability Using Designed Experiments	29
8.5	Destructive Degradation Data	29
8.6	An Alternative Degradation Data Model Using Stochastic Processes	30
8.7	Related Reading	30
8.8	Exercises for Chapter 8	31
9	Planning for Reliability Data Collection	31
9.1	Introduction	31
9.2	Planning Criteria, Optimization, and Implementation	32
9.2.1	Optimization in Planning	32
9.2.2	Implementing the Simulation-Based Framework	32
9.3	Planning for Binomial Data	32
9.4	Planning for Lifetime Data	32
9.5	Planning Accelerated Life Tests	32
9.6	Planning for Degradation Data	33
9.7	Planning for System Reliability Data	33
9.8	Related Reading	33
9.9	Exercises for Chapter 9	33
10	Assurance Testing	34
10.1	Introduction	34
10.1.1	Classical Risk Criteria	34
10.1.2	Average Risk Criteria	34
10.1.3	Posterior Risk Criteria	34
10.2	Binomial Testing	34
10.2.1	Binomial Posterior Consumer's and Producer's Risks	34
10.2.2	Hybrid Risk Criterion	35
10.3	Poisson Testing	35
10.4	Weibull Testing	35
10.4.1	Single Weibull Population Testing	36
10.4.2	Combined Weibull Accelerated/Assurance Testing	36
10.5	Related Reading	36
10.6	Exercises for Chapter 10	36
A	Acronyms and Abbreviations	37

B	Special Functions and Probability Distributions	377
B.1	Greek Alphabet	377
B.2	Special Functions	377
B.2.1	Beta Function	377
B.2.2	Binomial Coefficient	378
B.2.3	Determinant	378
B.2.4	Factorial	378
B.2.5	Gamma Function	378
B.2.6	Incomplete Beta Function	378
B.2.7	Incomplete Beta Function Ratio	378
B.2.8	Indicator Function	379
B.2.9	Logarithm	379
B.2.10	Lower Incomplete Gamma Function	379
B.2.11	Standard Normal Cumulative Density Function	379
B.2.12	Standard Normal Probability Density Function	379
B.2.13	Trace	379
B.2.14	Upper Incomplete Gamma Function	379
B.3	Probability Distributions	380
B.3.1	Bernoulli	380
B.3.2	Beta	380
B.3.3	Binomial	382
B.3.4	Bivariate Exponential	382
B.3.5	Chi-squared	383
B.3.6	Dirichlet	383
B.3.7	Exponential	386
B.3.8	Extreme Value	386
B.3.9	Gamma	389
B.3.10	Inverse Chi-squared	389
B.3.11	Inverse Gamma	392
B.3.12	Inverse Gaussian	392
B.3.13	Inverse Wishart	392
B.3.14	Logistic	396
B.3.15	Lognormal	396
B.3.16	Multinomial	399
B.3.17	Multivariate Normal	399
B.3.18	Negative Binomial	399
B.3.19	Negative Log-Gamma	401
B.3.20	Normal	403
B.3.21	Pareto	403
B.3.22	Poisson	403
B.3.23	Poly-Weibull	403
B.3.24	Student's t	406
B.3.25	Uniform	408
B.3.26	Weibull	408
B.3.27	Wishart	411

References 413

Author Index 427

Subject Index 431