This book gives a presentation of stochastic epidemic models and their statistical analysis. It focuses on simple epidemic models making use of modern probabilistic and statistical methods such as coupling, diffusion approximation, random graphs, likelihood theory for counting processes, martingales, the EM-algorithm, and MCMC methods. These methods are presented in a general form keeping the technical level at a minimum and then applied to epidemic models. The reader will learn about the theory of epidemic models and be introduced to many useful general techniques from probability and statistics. The lecture notes require an undergraduate-level knowledge of probability and statistics and is well suited for a one-semester graduate course.

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