

This book gives a self-contained introduction to the dynamic martingale approach to marked point processes (MPPs). Based on the notion of a compensator, this approach gives a versatile tool for analyzing and describing the stochastic properties of an MPP. In particular, the authors discuss the relationship of an MPP to its compensator and particular classes of MPPs are studied in great detail. The theory is applied to study properties of dependent marking and thinning, to prove results on absolute continuity of point process distributions, to establish sufficient conditions for stochastic ordering between point and jump processes, and to solve the filtering problem for certain classes of MPPs.

Although readers are assumed to be familiar with the basic notions of measure, integration, and probability theory, an appendix contains extensive surveys of the theory of conditional distributions and Lebesgue-Stieltjes calculus. Consequently researchers and graduate students in probability will find this an ideal introduction to this topic.

ISBN 0-387-94547-4



9 780387 945477

ISBN 0-387-94547-4



|                                                                    |           |
|--------------------------------------------------------------------|-----------|
| <b>Preface</b>                                                     | <b>v</b>  |
| <b>Basic Notation</b>                                              | <b>ix</b> |
| <b>1 Overview</b>                                                  | <b>1</b>  |
| 1.1 Introduction . . . . .                                         | 1         |
| 1.2 Definition of a Marked Point Process . . . . .                 | 3         |
| 1.3 Random Counting Measures . . . . .                             | 5         |
| 1.4 The Intensity Measure . . . . .                                | 6         |
| 1.5 The Point of Explosion . . . . .                               | 8         |
| 1.6 Point Process Filtrations and Stochastic Intensities . . . . . | 9         |
| 1.7 Two Examples of Martingales . . . . .                          | 13        |
| 1.8 The Compensator . . . . .                                      | 15        |
| 1.9 Conditional Distributions and Stopping Times . . . . .         | 20        |
| 1.10 An Explicit Formula for the Compensator . . . . .             | 22        |
| 1.11 Existence and Uniqueness . . . . .                            | 24        |
| 1.12 Absolute Continuity . . . . .                                 | 24        |
| 1.13 The Structure of Point Process Martingales . . . . .          | 25        |
| 1.14 Compensators with respect to General Filtrations . . . . .    | 26        |
| 1.15 Stochastic Ordering . . . . .                                 | 28        |
| 1.16 Filtering . . . . .                                           | 31        |

|          |                                                                                                                  |            |
|----------|------------------------------------------------------------------------------------------------------------------|------------|
| <b>2</b> | <b>Measurability Concepts for Stochastic Processes, Point and Jump Processes and Random Measures</b>             | <b>35</b>  |
| 2.1      | Measurability Concepts for Stochastic Processes . . . . .                                                        | 35         |
| 2.2      | Point Process Filtrations . . . . .                                                                              | 58         |
| 2.3      | Point Process Martingales . . . . .                                                                              | 71         |
| 2.4      | Measurability of Random Measures . . . . .                                                                       | 74         |
| 2.5      | Jump Process Filtrations . . . . .                                                                               | 82         |
| <b>3</b> | <b>Projections</b>                                                                                               | <b>89</b>  |
| 3.1      | Preliminaries . . . . .                                                                                          | 89         |
| 3.2      | Projections of Random Processes . . . . .                                                                        | 99         |
| 3.3      | Projections of Random Measures . . . . .                                                                         | 106        |
| 3.4      | Projections and Martingales . . . . .                                                                            | 108        |
| <b>4</b> | <b>The Compensator: Definition and Basic Properties</b>                                                          | <b>113</b> |
| 4.1      | Definition and Initial Examples . . . . .                                                                        | 113        |
| 4.2      | The Canonical Compensator . . . . .                                                                              | 127        |
| 4.3      | The First Marked Point after a Stopping Time . . . . .                                                           | 131        |
| <b>5</b> | <b>The Poisson Process</b>                                                                                       | <b>153</b> |
| 5.1      | Point Processes with Independent Increments . . . . .                                                            | 153        |
| 5.2      | Doubly Stochastic Poisson Processes . . . . .                                                                    | 163        |
| <b>6</b> | <b>The Marked Poisson Process</b>                                                                                | <b>175</b> |
| 6.1      | Doubly Stochastic Marked Poisson Processes . . . . .                                                             | 175        |
| 6.2      | Dependent Marking of Point Processes<br>with Application to the Marked Poisson Process . . . . .                 | 184        |
| 6.3      | Random Measures with Independent Increments;<br>Compound Poisson Processes . . . . .                             | 192        |
| <b>7</b> | <b>Continuous Time Markov Chains</b>                                                                             | <b>207</b> |
| 7.1      | Definition of a Markov Chain . . . . .                                                                           | 207        |
| 7.2      | Distribution of Markov Jump Processes . . . . .                                                                  | 211        |
| 7.3      | Construction of Markov Jump Processes . . . . .                                                                  | 222        |
| 7.4      | Minimal Solutions of Kolmogorov's Differential Equations . . . . .                                               | 226        |
| 7.5      | Markov Chains and Martingales . . . . .                                                                          | 231        |
| 7.6      | Martingale Characterization of Markov Chains . . . . .                                                           | 238        |
| 7.7      | Markov Modulated Poisson Processes . . . . .                                                                     | 239        |
| <b>8</b> | <b>Existence and Uniqueness<br/>of Point Process Distributions</b>                                               | <b>249</b> |
| 8.1      | Finite Dimensional Distributions; Uniqueness<br>of Point Process Distributions for a Given Compensator . . . . . | 249        |

|           |                                                                                       |            |
|-----------|---------------------------------------------------------------------------------------|------------|
| 8.2       | Existence of Point Process Distributions<br>for a Given Compensator . . . . .         | 258        |
| 8.3       | Stochastic Intensity Kernels . . . . .                                                | 262        |
| <b>9</b>  | <b>Stochastic Ordering</b>                                                            | <b>271</b> |
| 9.1       | Dependent Marking and Thinning<br>of Point Processes . . . . .                        | 271        |
| 9.2       | Representation of Jump Processes<br>by Marking and Thinning a Point Process . . . . . | 288        |
| 9.3       | Pathwise Comparison of Jump Processes . . . . .                                       | 293        |
| <b>10</b> | <b>Absolute Continuity</b>                                                            | <b>321</b> |
| 10.1      | Absolute Continuity of Internal Compensators . . . . .                                | 321        |
| 10.2      | Absolute Continuous Transformations<br>of Probability Measures . . . . .              | 332        |
| 10.3      | Representation of Point Process Martingales . . . . .                                 | 342        |
| <b>11</b> | <b>Filtering</b>                                                                      | <b>347</b> |
| 11.1      | Filtering of Partially Observed Point Processes . . . . .                             | 348        |
| 11.2      | Explicit Solutions of Filtering Equations . . . . .                                   | 367        |
| 11.3      | Filtering of Semi-Markov Processes . . . . .                                          | 377        |
| 11.4      | Filtering of Markov Chains . . . . .                                                  | 383        |
| 11.5      | Semi-Markov Modulated Poisson Processes . . . . .                                     | 389        |
| 11.6      | Nonlinear Filtering Equations . . . . .                                               | 395        |
| <b>A1</b> | <b>Monotone Class Theorems</b>                                                        | <b>401</b> |
| <b>A2</b> | <b>Kernels and Disintegration</b>                                                     | <b>405</b> |
| <b>A3</b> | <b>Conditional Probabilities<br/>and Conditional Distributions</b>                    | <b>409</b> |
| <b>A4</b> | <b>Lebesgue-Stieltjes Calculus</b>                                                    | <b>421</b> |
| A4.1      | Functions of Locally Bounded Variation . . . . .                                      | 421        |
| A4.2      | Lebesgue-Stieltjes Integral . . . . .                                                 | 422        |
| A4.3      | The Product and Exponential Formula . . . . .                                         | 425        |
| A4.4      | Two Facts from Renewal Theory . . . . .                                               | 430        |
| <b>A5</b> | <b>Random Times and Hazard Measures</b>                                               | <b>431</b> |
| A5.1      | Preliminaries . . . . .                                                               | 431        |
| A5.2      | The General Situation . . . . .                                                       | 434        |
| A5.3      | Hazard Measures of Marked Points . . . . .                                            | 440        |
| A5.4      | A Property of Conditional Random Times . . . . .                                      | 442        |

|                                                      |            |
|------------------------------------------------------|------------|
| <b>A6 Order Statistics</b>                           | <b>445</b> |
| A6.1 The Uncensored Case . . . . .                   | 445        |
| A6.2 Order Statistics with Censoring . . . . .       | 450        |
| <b>A7 Martingales</b>                                | <b>455</b> |
| <b>A8 Stochastic Ordering</b>                        | <b>457</b> |
| A8.1 Strassen's Theorem . . . . .                    | 457        |
| A8.2 Weak Convergence . . . . .                      | 459        |
| A8.3 A Comparison Result for Marked Points . . . . . | 460        |
| <b>Bibliographical Comments</b>                      | <b>465</b> |
| <b>References</b>                                    | <b>471</b> |
| <b>Symbol Index</b>                                  | <b>481</b> |
| <b>Subject Index</b>                                 | <b>487</b> |